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# (In)Efficient Bargaining in the Family

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**Abstract**

This paper describes how families bargain to reach an agreement recognizing that the negotiation process is costly and often difficult. Our focus is not only on the efficient outcomes of the decision process but also on the bargaining process. We propose an evolutionary bargaining approach that identifies who is willing to make a concession depending on the perceived cost of bargaining failure. The theoretical analysis extends the original Nash-Harsanyi cardinal representation to ordinal preferences and rationalizes agreements that can be inefficient. Implications for efficiency and income distribution are discussed. We illustrate the usefulness of our theory in an empirical application.

**Keywords:** Bargaining agreements, household efficiency, intra-household welfare, threat strategies.

**JEL Classification:** D13, D61, D74.

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## 1. Introduction

The family is a basic institution in human society. Yet, it has evolved over time and across cultures and countries (Kertzer, 1991; Jonas, 2007). The functioning of the family involves negotiations among household members about individual time allocation, the management of household resources, and the intra-household distribution of income and purchasing power. However, the negotiations are often complex and deal with conflicts of interest among family members.

Most economic models of the family assume a stable institutional environment where efficient outcomes are the optimal solutions of an intra-household decision process (e.g., Chiappori, 1988; Chiappori, Donni and Komunjer, 2012; Chiappori and Meghir, 2014). But there is empirical evidence of inefficiency in household decisions (e.g., Udry, 1996; Campbell, 2006), reflecting that household members have difficulties identifying and implementing efficient agreements. These difficulties arise under asymmetric information (e.g., Myerson and Satterthwaite 1983) as well as bounder rationality (e.g., Simon, 1979; Rubinstein, 1998; Oprea, 2020). This suggests a need to investigate how family members bargain with each other, what is the nature of bargaining agreements, and how the outcome of the bargaining process relates to efficiency and welfare distribution within a household. This study addresses these issues, with a focus on the intra-household negotiation process in a family environment that involves both cooperation and conflicts when dividing household resources and the fruits of cooperation among the members (Sen, 1990). The paper contributes to the economics of the household in several ways. We develop a general model of the intra-household bargaining process represented by a sequence of offers. We evaluate how the sequence convergences to a bargaining agreement. The approach allows for a joint analysis of bargaining, inefficiency, and income distribution within the household. And the model is empirically tractable, its usefulness being illustrated in an application to a sample of Italian households where we identify individual welfares of wives and husbands implementing a collective household model.

This paper is about bargaining in the family. We develop a joint analysis of bargaining and efficiency applied to household economics. We allow for inefficiency, reflecting that attaining efficiency may not succeed when bargaining is costly or difficult. Such difficulties can arise under bounded rationality (Simon, 1979), as the result of cognitive and non-cognitive limitations, bargaining cost, information asymmetries, commitment frictions or time constraints (de Palma et al., 1994; Oprea, 2020). In these cases, bargaining can lead to inefficient outcomes. We propose to represent the intra-household bargaining process by an evolutionary game where family members negotiate considering iterative offers. Evolutionary strategies have three attractive characteristics: 1) they provide realistic representations of decision making; 2) they can capture the presence of bounded rationality, reflecting difficulties individuals have in obtaining and processing information in the bargaining process; 3) they leave open the possibility of converging to efficient allocations (Gintis, 2007; Sandholm, 2010; Chavas and Wang, 2021). We propose measuring bargaining strength by each individual's willingness-to-pay to avoid bargaining failure. At each step of the bargaining process, our evolutionary scheme identifies who is willing to make a concession depending on the relative perceived cost of bargaining failure. In this context, we examine how the bargaining process converges to bargaining agreements, which may or may not be efficient. We examine the conditions under which bargaining agreements would be efficient. We also evaluate the implications of bargaining agreements for income and welfare distribution within the household.

Our analysis builds on Nash bargaining following the seminal contribution made by Nash (1950, 1953) and Harsanyi (1950, 1963, 1977).[5] Previous literature has seen many applications of the Nash bargaining model to intra-household decisions, including McElroy and Horney (1981), McElroy (1990), Notburga (1992), Lundberg and Pollak (1993, 1996, 2003), Donni and Ponthieux (2011) and Chiappori, Donni and Komunjer (2012). Like the Nash bargaining model,[6] our analysis uses bargaining failure as key determinants of individual bargaining power. But we extend the Nash bargaining model in two important directions: 1/ under bounded rationality, our evolutionary scheme defines bargaining agreements that are not necessarily efficient; and 2/ unlike the Nash bargaining model,[7] our analysis is presented under ordinal preferences and general household technology.

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[5] There are many approaches to the economic analysis of bargaining (e.g., Roth, 1985). The Nash-Harsanyi bargaining model focuses on cooperative bargaining. This differs from the non-cooperative approach to bargaining explored by Rubinstein (1982, 1998). While the two approaches can generate similar outcomes (Binmore et al., 1986), we note that bounded rationality means that individuals can have difficulties evaluating efficiency as well as strategic behavior (including coalition formation). Also, the non-existence of core allocations (i.e., allocations that are "coalition-proof") can be problematic (Forges et al., 2002). Our representation of the bargaining process as a sequence of (possibly inefficient) offers avoids these issues.

[6] In the Nash bargaining game, bargaining failure defines threat points that provide lower bounds on individual welfare under bargaining agreements (Nash, 1950, 1953). As argued by Zeuthen (1930) and Harsanyi (1977), these threat points also identify who is more willing to make concessions in the bargaining process.

[7] The basic Nash-Harsanyi bargaining model relies on cardinal preferences under the expected utility model (Nash, 1950, 1953). Extensions of this model under non-expected utility models have been explored by Rubinstein et al. (1992) and Hanany and Safra (2000). Our analysis is more general: it applies under general ordinal preferences.

We evaluate how an evolutionary bargaining scheme converges to bargaining agreements. These bargaining agreements can be inefficient (reflecting bounded rationality). Our analysis then provides a measure of the welfare loss from bargaining inefficiency. Alternatively, bargaining agreements would be efficient if the bargaining process is costless. In this case, the bargaining outcome identifies a unique point on the Pareto utility frontier. Our analysis also explores the implications of the bargaining process for income and welfare distribution. We show how the intra-household income distribution depends on the threat points representing bargaining failure. This reflects the positive aspect of our analysis: income distribution can vary from egalitarian (when all individuals have the same purchasing power at the threat points) to very unequal (when bargaining failure is associated with high inequality). Finally, we show that the outcome of the bargaining process can be represented by the maximization of a "generalized Nash product" that identifies the set of contracts representing intra-household bargaining agreements. This representation provides a convenient framework to support empirical analyses of household behavior.

We illustrate the usefulness of our theory in an empirical application based on a sample of Italian family enterprises that simulates the proposed iterative bargaining scheme, The empirical analysis shows that most of the family agreements are inefficient, lending empirical support to Simon's hypothesis that rational individuals can be sufficiently satisfied also at inefficient but less conflictual positions. We also investigate the factors affecting agreeableness, the difficulty in reaching an agreement, and costs yielded by inefficiency. The theoretical as well as the empirical results are new contributions to the literature.

## 2. Household Efficiency

This section establishes the notation. Building on previous literature, it presents a characterization of household efficiency that sets the stage for the analysis given in the rest of the paper. Consider a household composed of $n$ members, with $n \geq 2$. Household members make decisions related to both consumption and production activities. They choose consumption goods, including public consumption market goods $x_0 = (x_{01}, x_{02}, \dots)$ (i.e., purchased goods that affect the welfare of multiple household members, such as having a roof in the house, or someone smoking in the house), private consumption market goods $x_i = (x_{i1}, x_{i2}, \dots), i \in N \equiv \{1, \dots, n\}$ (e.g., clothing and food),[8] and non-market goods $z = (z_1, z_2, \ldots)$ (e.g., parental care). The inclusion of public goods $x_0$ and non-market goods $z$ captures situations where there are externalities across household members (Ellickson, 2008). The household also chooses production goods, including market goods $y$ used to generate household income (e.g., wage labor or inputs and outputs in farms households). The netput notation is used for the market goods $y = (y_1, y_2, \ldots)$: $y_i \leq 0$ when the $i$-th good is an input and $y_j \geq 0$ when the $j$-th good is an output.

Preferences of the $i$-th member of the household are represented by the ordinal utility function $u_i(x_0, x_i, z), i \in N$. Let $F$ be the feasible set for household production, where $(y, z) \in F$ means that $(y, z)$ is feasible. Let $X_0$ be the feasible set of $x_0$ and $X_i$ be the feasible set of $x_i, i \in N$. And let $p_0 > 0$ be the prices of $x_0$, $p_i > 0$ be the prices of $x_i, i \in N$, and $q = (q_1, q_2, \ldots) > 0$ be the prices of $y$.

The household receives income $[q\ y] = \sum_j q_j y_j$. Household income can come from three sources: from wage labor; from profit, generated by household production activities involving marketed goods (e.g., farm profit in farm household); and from non-labor sources (e.g., interests, dividends, and rents). The allocation of time by each household member involves three components: wage labor, treated as a market good and included in the vector $y$, leisure time included in the vector of private market goods, and work spent on household production activities included in the vector $z$, where the sum of these three-time components is equal to the total amount of time available for each individual. These time allocation decisions allow for specialization of tasks related to both intra-household work and labor market participation (Becker, 1960, 1981; Cigno, 1991).

Throughout the paper, we examine the economics of household decisions about the allocation $(x, y, z)$, where $x \equiv (x_0, x_1, \ldots, x_n) \in X \equiv X_0 \times X_1 \times \ldots \times X_n$ and $(y, z) \in F$ under the following assumptions:

Assumption A1: The utility function of individual $i$, $u_i(x_0, x_i, z)$, is continuous and quasi-concave in $(x_0, x_i, z), i \in N$.

[8] Clothing and food are private goods whose consumption is assignable to specific household members. While food items are consumed individually, food purchases are often recorded only at the household level.

Assumption A2: For each $i \in N$, the utility function $u_i(x_0, x_i, z)$ is non-satiated in $x_i$, meaning that for any $x_i \in X_i$ and any $(x_0, z)$, there is a $x_i' \in X_i$ satisfying $x_i' \geq x_i$, $x_i' \neq x_i$ and $u_i(x_0, x_i', z) > u_i(x_0, x_i, z)$.

Assumption A3: The sets $X_0, X_1, \ldots, X_n$ are closed and convex, each set has a lower bound, and the set $F$ is closed.

Assumptions A1 and A2 are standard. For each individual, the utility function $u_i(x_0, x_i, z)$ is ordinal and defined up to a monotonic increasing transformation. This means that utilitarian considerations and interpersonal comparison of utilities are superfluous. Note the generality of the approach. First, A3 does not assume that the set $F$ is necessarily convex. This allows for a general household production technology because we do not assume that the technology exhibits constant returns to scale. Second, the allocation $(x, y, z)$ can occur over time, allowing for an intertemporal analysis. In this case, the vector $y$ would include saving/investment (or borrowing if negative) that transfers income across periods and the utility functions would reflect time preferences. Third, the analysis can apply under uncertainty. Representing the uncertainty by states of nature, the elements of the vector $(x, y, z)$ would then be defined as being state-dependent and the utility functions would represent risk preferences (Debreu, 1959; Radner, 1968).

Our analysis will rely in part on the classical concept of Pareto efficiency. We define a household allocation to be efficient if there does not exist another feasible allocation that can make one household member better off without making anyone else worse off. Feasibility includes the household budget constraint

$$p_0 x_0 + \sum_{i \in N} p_i x_i \leq q\, y, \tag{1}$$

stating that household expenditures $[p_0 x_0 + \sum_{i \in N} p_i x_i]$ cannot exceed household income $[q\, y]$. Letting $p = (p_0, p_1, \ldots, p_n)$, consider the maximization problem

$$W(p, q, U) = Max_{x,y,z}\{q\, y - p_0 x_0 - \sum_{i \in N} p_i x_i : u_i(x_0, x_i, z) \geq U_i, i \in N; \\ x \in X, (y, z) \in F\}, \tag{2}$$

where $U_i$ is a reference utility level for the $i$-th household member, $i \in N$, and $U = (U_1, \ldots, U_n)$. Denote the solution to (2) by $x^*(p, q, U), y^*(p, q, U)$ and $z^*(p, q, U)$. The allocation evaluated in equation (2) is conditional on $U = (U_1, \ldots, U_n)$. We make the additional assumption:

Assumption A4: $U \in \mathcal{U}$ where $\mathcal{U} \equiv \{U : W(p, q, U) \geq 0; [x_i : x_i < x_i^*(p, q, U)] \cap X_i \neq \emptyset, i \in N\}$.

Assumption A4 allows for a wide income distribution, including poverty where low purchasing power is associated with low consumption of private goods, but it rules out situations of complete destitution where $x_i^*(p, q, U)$ would be located on the lower bound of $X_i$. This ensures that each individual has enough purchasing power to allow a reduction in consumption of private goods, thus guaranteeing that individual willingness-to-pay remains a valid welfare measure. Assuming that the set $\mathcal{U}$ is not empty, consider the following choice for $U$:

$$U^* \in \{U: W(p, q, U) = 0, U \in \mathcal{U}\}. \tag{3}$$

A representation of household efficiency is presented next. See the proof in Appendix A.

Proposition 1. Under assumptions A1-A4, a household allocation $(x^*, y^*, z^*)$ is Pareto efficient if and only if it satisfies equations (2) and (3).

For a given household welfare distribution $U = (U_1, \dots, U_n)$, equation (2) associates an efficient allocation with the maximization of household net income $[q\ y - p_0 x_0 - \sum_{i \in N} p_i\ x_i]$. It defines $W(p, q, U)$ as the maximized household net income conditional on $U = (U_1, \dots, U_n)$. Equation (3) states that the distribution of welfare $U = (U_1, \dots, U_n)$ is such that the maximized net income $W(p, q, U)$ is entirely redistributed among the $n$ household members. To see that (3) is an integral part of an efficient allocation, note that having $[q\ y] < [p_0 x_0 + \sum_{i \in N} p_i\ x_i]$ would not be feasible (as it would not satisfy the budget constraint (1)) while having $[q\ y] > [p_0 x_0 + \sum_{i \in N} p_i\ x_i]$ would be inefficient (under assumption A2), implying the household budget constraint (1) must be binding under efficiency.

A useful interpretation of equation (3) is that choosing $U = (U_1, \dots, U_n)$ such that $W(p, q, U) = 0$ defines the household utility frontier. For example, when $n = 2$, solving $W(p, q, U_1, U_2) = 0$ for $U_2$ gives the household utility frontier $U_2(p, q, U_1)$, which traces out the utilities $\{U_1, U_2(p, q, U_1)\}$ that can be attained under an efficient household allocation. Thus, when $n \geq 2$, equations (2) and (3) identify the set of efficient allocations along the household utility frontier. It makes it clear that efficient allocations are not unique (as they change depending on the values taken by $U_1$ and $U_2(p, q, U_1)$). Which point on the utility frontier is obtained depends on the distribution of welfare within the household. In turn, since $x^*, y^*$ and $z^*$ depend on $U^*$, the welfare levels obtained by each household member will affect behavior.

Next, we establish linkages between Proposition 1 and previous literature on household economics. Proposition 1 can alternatively be written as follows.

Corollary 1. Under assumptions A1-A4, a household allocation $(x^*, y^*, z^*)$ is Pareto efficient if and only if it satisfies equation (3), along with

$$e_i(p_i, x_0, z, U_i) = \min_{x_i}\{p_i x_i : u_i(x_0, x_i, z) \geq U_i, x_i \in X_i\}, i \in N, \quad (4)$$

$$E(p, z, U) = \min_{x_0}\{p_0 x_0 + \sum_{i \in N} e_i(p_i, x_0, z, U_i) : x_0 \in X_0\}, \quad (5)$$

$$\pi(q, z) = \max_y\{q\, y : (y, z) \in F\}, \quad (6)$$

$$W(p, q, U) = \max_z\{\pi(q, z) - E(p, z, U)\}. \quad (7)$$

Equations (4)-(7) follow directly from a stage-wise decomposition of the optimization problem in (2). Conditional on $(x_0, z)$, equation (4) is a standard expenditure minimization problem. For the $i$-th member of the household, it generates the private expenditure function $e_i(p_i, x_0, z, U_i)$. Denoting by $w_i$ the part of household income received by the $i$-th individual, a dual representation of (4) is given by the standard utility maximization problem $V_i(p_i, x_0, z, w_i) = \max_{x_i}\{u_i(x_0, x_i, z) : p_i x_i \leq w_i, x_i \in X_i\}$, where $V_i(p_i, x_0, z, w_i)$ is an indirect utility function satisfying $w_i = e_i(p_i, x_0, z, V_i(p_i, x_0, z, w_i)), i \in N$ (Deaton and Muellbauer, 1980: 38). Because $z$ are non-market goods, they have shadow prices. Under differentiability, $\partial e_i / \partial z$ measures the shadow prices of $z$ for each $i \in N$. Equation (5) is an optimization problem involving the choice of $x_0$ conditional on $z$ (Pollak and Wachter, 1975). It generates the household expenditure function $E(p, z, U)$. Equation (6) is a standard profit maximization problem, yielding the profit or household income function $\pi(q, z)$. Under differentiability, $\partial \pi / \partial z$ measures the shadow cost of the non-market goods $z$. Equation (7) states that the efficient choice of non-market goods $z$ is consistent with the maximization of net aggregate household income $[\pi(q, z) - E(p, z, U)]$. The choices for $(x_0, y, z)$ given in (5)-(7) include the case of efficient management of externalities across individuals within the household (Coase, 1960; Ellickson, 2008; Chavas, 2015).

From equation (7), feasibility means that the household budget constraint can be written as $E(p, z, U) \leq \pi(q, z)$ or $W(p, q, U) \geq 0$. From equation (3), Pareto efficiency means that $E(p, z, U) = \pi(q, z)$, implying that the household budget constraint is necessarily binding. If $W(p, q, U) \geq 0$ under feasibility and $W(p, q, U) = 0$ under Pareto efficiency, it follows that $W(p, q, U) > 0$ is necessarily associated with inefficiency. The inefficiency can come from three sources: 1/ the household does not maximize profit, thus lowering its purchasing power; 2/ the consumers do not minimize their expenditures; 3/ some of the income is not used, with adverse effects on expenditures and household welfare.

The efficiency considerations presented in Proposition 1 and Corollary 1 are commonly assumed in the analysis of economic behavior (Deaton and Muellbauer, 1980). However, empirical research has presented evidence of situations where families are unable to reach efficient resource allocations (Udry 1996; Duflo and Udry 2004; Apps and Rees, 2010). Next, we investigate how the bargaining process can affect intra-household allocations.

## 3. Intra-household Bargaining

How to choose a point $U^*$ on the Pareto utility frontier, as given in equation (3)? Many efficient points exist along the Pareto utility frontier. Given $w_i = e_i(p_i, x_0, z, V_i(p_i, x_0, z, w_i))$, moving along the Pareto utility frontier is associated with changing the distributions of income $(w_1, \dots, w_n)$ within the household. In egalitarian households, we would have $w_1 = w_2 = \cdots = w_n$. However, efficiency can also be associated with unequal distribution of income. Under extreme inequality, it is possible to have an efficient allocation where one individual receives most of the household income. The sharing of income within the household can be measured by the $i$-th individual sharing rule $s_i = w_i / \sum_{i \in N} w_i$, $i \in N$. A very unequal income distribution would occur when $s_i$ is large and close to 1 for one individual. Although this would likely imply a very unequal intra-household distribution of welfare, this situation can be consistent with Pareto efficiency. For example, it could happen when an individual acquires enough power within the household to shift the welfare distribution in her/his favor (Crockett et al., 2011). This indicates a need to go beyond Pareto efficiency and to examine the exercise of power and its implications for income and welfare distribution within the household.

This section investigates intra-household distribution as the outcome of a bargaining process among the $n$ members of the household, which could lead to either efficient or inefficient outcomes. The analysis is inspired by seminal work on bargaining by Nash (1950) and Harsanyi (1950, 1963, 1977). As noted in previous literature, the Nash-Harsanyi approach is limiting as it was presented under the expected utility model where utilities are defined up to a positive linear transformation. We extend the Nash-Harsanyi approach by presenting our analysis under ordinal preferences.

The analysis starts with threat points. Threat points are defined as feasible allocations representing outcomes that would arise in case of bargaining failure among the $n$ household members. We denote the threat allocation for the $i$-th individual by $(x_{0i}^t, x_i^t, y_i^t, z_i^t) \in X_0 \times X_i \times F, i \in N$. Denote by $U_i^t = u_i(x_{0i}^t, x_i^t, z_i^t)$ the utility obtained by individual $i \in N$ if no agreement is reached, with threat utilities given by $U^t = (U_1^t, \dots, U_n^t)$. In this section, we start with the simple situations where the threat points are treated as given. How bargaining can fail and how the threat points get determined will be discussed in Section 6.

Using equation (4), $e_i(p_i, x_0, z, U_i)$ measures the private expenditure (or purchasing power) of the $i$-th individual conditional on $(x_0, z, U_i)$. Let

$$\Delta_i(p_i, x_0, x_i, z, U_i^t) = e_i(p_i, x_0, z, u_i(x_0, x_i, z)) - e_i(p_i, x_0, z, U_i^t),\ i \in N. \quad (8)$$

The term $\Delta_i(p_i, x_0, x_i, z, U_i^t)$ in (8) is a willingness-to-pay measure for the $i$-th individual facing a choice between $(x_i, x_0, z)$ and the threat point $U_i^t = u_i(x_0^t, x_i^t, z^t), i \in N$. For a given $(x_i, x_0, z)$, it is a money-metric measure of the welfare loss perceived by the $i$-th individual facing the prospect of reaching the breakdown position: the greater $\Delta_i(p_i, x_0, x_i, z, U_i^t)$, the greater the reduction in her/his purchasing power, and the greater the associated welfare loss. Given $U_i^t = u_i(x_{0i}^t, x_i^t, z_i^t)$, note that the individual willingness-to-pay measure $\Delta_i(p_i, x_0, x_i, z, U_i^t)$ is invariant to a positive monotonic transformation of $u_i$.

To the extent that the threat outcomes are feasible and always available, no individual will not be willing to accept any payoff less favorable than his/her threat point. Thus, we restrict our bargaining analysis to the allocations $(x, y, z) \in X \times F$ satisfying $u_i(x_0, x_i, z) \geq U_i^t, i \in N$. It follows from (8) that $\Delta_i(p_i, x_0, x_i, z, U_i^t) \geq 0, i \in N$, stating that the willingness-to-pay to avoid the threat points is non-negative. It also implies that $W(p, q, U^t) \geq 0$, with $W(p, q, U^t)$ providing a measure of the aggregate welfare loss associated with bargaining failure.

We propose to use $\Delta_i(p_i, x_0, x_i, z, U_i^t)$ to evaluate the bargaining position of the $i$-th individual, $i \in N$.

Definition 1: The $i$-th individual facing $(x_0, x_i, z)$ is less willing to accept a bargaining failure than individual $i'$ if

$$\Delta_i(p_i, x_0, x_i, z, U_i^t) > \Delta_{i'}(p_{i'}, x_0, x_{i'}, z, U_{i'}^t), i \in N, i' \in N - i. \quad (9)$$

Definition 1 states that a move from an allocation $(x, y, z) \in X \times F$ to the threat points would decrease the purchasing power more for individual $i$ than individual $i'$. It asserts that the $i$-th individual is less willing to face a bargaining failure (compared to $i'$) when a bargaining failure, represented by a move from $(x_0, x_i, z)$ to $(x_{0i}^t, x_i^t, z_i^t)$, would reduce his/her purchasing power more than individual $i'$. For a given $(x_0, x_i, z)$, the greater $\Delta_i(p_i, x_0, x_i, z, U_i^t)$ is, the less the individual is willing to accept a bargaining failure.

Definition 2: A feasible allocation $(x, y, z) \in X \times F$ satisfying equation (1) is a bargaining agreement for all $n$ individuals if

$$\Delta_i(p_i, x_0, x_i, z, U_i^t) = M(p, x, z, U^t) \text{ for all } i \in N, \quad (10a)$$

where

$$M(p, x, z, U^t) = \max_i \{\Delta_i(p_i, x_0, x_i, z, U_i^t): i \in N\}. \quad (10b)$$

Definition 2 states that a feasible allocation $(x, y, z)$ is a bargaining agreement when all $n$ individuals are equally willing to accept a bargaining failure. How can household members proceed to reach a bargaining agreement? Could bargaining agreements be inefficient? Under what conditions would an agreement be efficient? Building on evolutionary games (Sandholm, 2010), our analysis provides answers to these questions. We focus on the following iterative scheme providing a representation of the bargaining process under bounded rationality.

Bargaining scheme S:

- Step S1: Start at iteration $k = 1$. Propose a feasible allocation $(x^k, y^k, z^k) \in X \times F$ satisfying equation (1), with corresponding utilities $U_i^k = u_i(x_0^k, x_i^k, z^k), i \in N$. Let $\Delta_i^k = [e_i(p_i, x_0^k, z^k, U_i^k) - e_i(p_i, x_0^k, z^k, U_i^t)]$, $i \in N$, and $M^k = \max_i \{\Delta_i^k: i \in N\}$.
- Step S2: Find the set $N^k$ of individuals being least willing to face a failure

  $$N^k = \{i: \Delta_i^k = M^k, i \in N\}. \quad (11)$$
- Step S3:
  - step S3a: If $N^k = N$, then stop the bargaining process at $k^\# = k$ and announce $(x^{k^\#}, y^{k^\#}, z^{k^\#})$ as a bargaining agreement.
  - step S3b: If $N^k \neq N$, propose a feasible allocation $(x^{k+1}, y^{k+1}, z^{k+1}) \in X \times F$ satisfying equation (1),

    $$\Delta_i^{k+1} < \Delta_i^k, i \in N^k, \text{ and} \quad (12a)$$

    $$\Delta_i^{k+1} \leq M^k, i \in N - N^k, \quad (12b)$$

    where $U_i^{k+1} = u_i(x_0^{k+1}, x_i^{k+1}, z^{k+1}), \Delta_i^{k+1} = [e_i(p_i, x_0^{k+1}, z^{k+1}, U_i^{k+1}) - e_i(p_i, x_0^{k+1}, z^{k+1}, U_i^t)]$, $i \in N$, and $M^{k+1} = \max_i \{\Delta_i^{k+1}: i \in N\}$. Then, let $k = k + 1$ and go to step S2.

The bargaining scheme S involves identifying the individuals who are the least willing to face a bargaining failure. At the $k$-th iteration, the set of these individuals is denoted by $N^k$ in equation

(11) in step S2. In a way consistent with Definition 2, step S3a identifies a bargaining agreement occurring when $N^k = N$, i.e. when all $n$ individuals are equally willing to face a bargaining failure. In step S3b, equations (12) state that the individuals in $N^k$ are the ones making a concession at the $k$-th iteration, where making a concession means reducing their willingness to face a bargaining failure. As in Zeuthen (1930), the individuals making concessions are the ones least willing to face a bargaining failure (Zeuthen, 1930; Harsanyi, 1950, 1977). The process of making concessions in S leads to a bargaining agreement (see the proof in Appendix A).

Proposition 2: Upon convergence, the bargaining scheme S identifies a bargaining agreement $(x^{k^\#}, y^{k^\#}, z^{k^\#})$.

Proposition 2 identifies a bargaining agreement. Note that Proposition 2 does not say that a bargaining agreement is unique. In general, it allows for multiple bargaining agreements. From Definition 2, a bargaining agreement occurs when all household members are equally willing to face a bargaining failure.

This raises two questions. First, what are the relationships between a bargaining agreement and bargaining power? Second, what are the linkages between bargaining agreements and Pareto efficiency? These questions are addressed next.

## 4. Bargaining Position and Income Distribution

From equations (8)-(10), our definition of a bargaining agreement relies on the $i$-th individual willingness-to-pay measure $\Delta_i(p_i, x_0, x_i, z, U_i^t) \equiv [e_i(p_i, x_0, z, u_i(x_0, x_i, z)) - e_i(p_i, x_0, z, U_i^t)], i \in N$. This measure depends on individual preferences $u_i(x_0, x_i, z)$ as well as the threat utility $U_i^t = u_i(x_0^t, x_i^t, z^t), i \in N$. Our analysis indicates that any factor that contributes to increasing (decreasing) $\Delta_i(p_i, x_0, x_i, z, u_i(x_0^t, x_i^t, z^t))$ would increase (decrease) the $i$-th individual's willingness to make a concession and thus weaken (strengthen) his/her bargaining position within the household. This section discusses the implications of these arguments.

First, consider the role of preferences and their effects on bargaining outcomes. An example is the role of patience in situations where $(x, y, z)$ represents a multi-period allocation over some future planning horizon. Different individual may discount the future at different rates, with "more patient" individuals having a lower discount rate (Rubinstein, 1982). A result we obtain from our analysis is that greater patience would weaken the bargaining position of individual $i$ when it induces an increase in $\Delta_i(p_i, x_0, x_i, z, u_i(x_0^t, x_i^t, z^t))$, as this would make individual $i$ more willing to make a concession. This would occur when bargaining failure has large adverse effects on future benefits, thus associating greater patience with a rise in *ex ante* willingness to pay to avoid a bargaining failure. This result differs from the one obtained by Rubinstein (1982), reflecting that our evaluation of bargaining is relative to the threat points.

Another example involves the effects of risk preferences when benefits are uncertain. How would change in risk aversion affect the outcome of the bargaining process (e.g., Murnighan et al., 1988)? From our analysis, the willingness-to-pay $\Delta_i(p_i, x_0, x_i, z, u_i(x_0^t, x_i^t, z^t))$ is an *ex ante* measure. For risk averse individuals, this willingness-to-pay would decline when benefits become more uncertain. A prediction from our analysis is the $i$-th individual becoming more risk averse would weaken his/her bargaining position when this change raises $\Delta_i(p_i, x_0, x_i, z, u_i(x_0^t, x_i^t, z^t))$, thus increasing his/her willingness to make a concession. This would occur when bargaining failure tends to increase individual risk exposure, thus associating higher risk aversion with a rise in *ex ante* willingness to pay to avoid a bargaining failure.

Perhaps more importantly, our analysis provides useful insights on the role of threat points (represented by $U_i^t = u_i(x_0^t, x_i^t, z^t), i \in N$) and their effects on intra-household bargaining and income distribution. For the $i$-th individual, the expenditure function $e_i(p_i, x_0, z, U_i)$ reflects his/her purchasing power under utility $U_i$ and can be used to evaluate how income is distributed within the household. To see that, consider the following expenditure share associated with the $i$-th household member

$$\sigma_i(p_i, x_0, z) = \frac{e_i(p_i, x_0, z, u_i(x_0, x_i, z))}{\sum_{i' \in N} e_{i'}\left(p_{i'}, x_0, z, u_{i'}(x_0, x_{i'}, z)\right)} \in [0, 1], i \in N. \tag{13}$$

Equation (13) measures the proportion of expenditures on private goods captured by the $i$-th individual. This measure has several attractive features. First, it reflects the role of relative bargaining power within the household. Intuitively, seeing a family member receiving a larger (smaller) expenditure share would indicate that this individual has a greater (weaker) bargaining power. Second, it is a money metric measure of how income is shared within the household, thus providing useful insights into intra-household welfare distribution. Finally, as illustrated below, $\sigma_i(p_i, x_0, z)$ in equation (13) is empirically tractable.

What are the linkages between $\sigma_i(p_i, x, z)$ in equation (13) and a bargaining agreement? To answer this question, let $(x^b, y^b, z^b)$ be a bargaining agreement satisfying Definition 2. It follows from equations (8)-(10) that $e_i\left(p_i, x_0^b, z^b, u_i\left(x_0^b, x_i^b, z^b\right)\right) - e_i\left(p_i, x_0^b, z^b, U_i^t\right) = k, i \in N$, for some constant $k$, implying that

$$e_i\left(p_i, x_0^b, z^b, u_i\left(x_0^b, x_i^b, z^b\right)\right) - e_i\left(p_i, x_0^b, z^b, U_i^t\right) =$$

$$\sum_{i \in N}\left[e_i\left(p_i, x_0^b, z^b, u_i\left(x_0^b, x_i^b, z^b\right)\right) - e_i\left(p_i, x_0^b, z^b, U_i^t\right)\right]/n,$$

or, using equation (13),

$$\sigma_i\left(p_i, x_0^b, z^b\right) = \frac{1}{n} + \frac{e_i(p_i, x_0^b, z^b, U_i^t) - [\sum_{i' \in N} e_{i'}(p_{i'}, x_0^b, z^b, U_{i'}^t)/n]}{\sum_{i' \in N} e_{i'}\left(p_{i'}, x_0^b, z^b, u_{i'}\left(x_0^b, x_{i'}^b, z^b\right)\right)}. \quad (14)$$

Equation (14) shows how the expenditure share of the $i$-th individual depends on the threat points $(U_1^t, \ldots, U_n^t)$. It indicates that a *ceteris paribus* increase in $U_i^t$ would increase both $e_i\left(p_i, x_0^b, z^b, U_i^t\right)$ and $\sigma_i\left(p_i, x_0^b, z^b\right)$. Thus, an individual would increase his/her expenditure share when he/she can make a bargaining failure less threatening. Alternatively, an individual would receive a lower expenditure share when faced with more threatening bargaining failures. Associating the threat utility $U_i^t$ with the strength of the $i$-th individual's bargaining power, we obtain the intuitive result: a family member with stronger bargaining power would capture a larger share of household expenditures.

Equation (14) implies that, under a bargaining agreement, $\sigma_i\left(p_i, x_0^b, z^b\right)\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 1/n$ when $e_i\left(p_i, x_0^b, z^b, U_i^t\right)\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}\sum_{i' \in N} e_{i'}(p_{i'}, x_0^b, z^b, U_{i'}^t)/n$. It follows that all individuals would have equal expenditure share (with $\sigma_i = 1/n$) when they have the same purchasing power at the threat points (when bargaining fails). This corresponds to an egalitarian distribution of income where each household member receives the same individual income. Our analysis also allows for unequal income distribution. Indeed, equation (14) can generate a very unequal distribution of household income when purchasing power varies a lot across household members under bargaining failure. In this case, individuals facing damaging threats (with low $e_i\left(p_i, x_0^b, z^b, U_i^t\right)$) would receive lower income, while individuals with high $e_i\left(p_i, x_0^b, z^b, U_i^t\right)$ would receive a larger share of household income. These results have two implications. First, they illustrate that our analysis is positive (and not normative). As such, our approach can provide useful insights into intra-household welfare inequality and distribution issues (including the case of domestic violence). Second, our analysis stresses the importance of threat points as they affect both bargaining power and the distribution of income. The socio-economic and political factors affecting threat strategies are discussed in Section 6 below.

## 5. Bargaining and Efficiency

Note that Proposition 2 does not require the bargaining agreement $(x^{k^{\#}}, y^{k^{\#}}, z^{k^{\#}})$ to be Pareto efficient. Thus, it allows for inefficient bargaining outcomes. This seems useful to the extent that assessing household efficiency typically requires a lot of information about all aspects of household allocations. When obtaining and processing this information proves difficult, identifying and implementing efficient allocations may be problematic in household bargaining.

Alternatively, if the bargaining outcome $(x^{k^{\#}}, y^{k^{\#}}, z^{k^{\#}})$ is not Pareto efficient, it means the possibility of untapped efficiency gains. Letting $U^{\#} = \left(U_1^{k^{\#}}, \dots, U_n^{k^{\#}}\right)$, the corresponding aggregate efficiency gain would be $W(p, q, U^{\#}) > 0$. This raises the question: could the bargaining scheme S be modified to converge to an efficient allocation? A positive answer to this question is presented next.

Bargaining scheme S^E (Efficient scheme): The scheme S^E is the same as S except that step S3 is replaced by S^E3:

- Step S^E3
  - step S^E3a: If $N^k = N$ and $W(p, q, U^k) = 0$, then stop the bargaining process at $k^* = k$ and announce $(x^{k^*}, y^{k^*}, z^{k^*})$ as a bargaining agreement.
  - step S^E3b: If $N^k \neq N$ or $W(p, q, U^k) > 0$, propose a feasible allocation $(x^{k+1}, y^{k+1}, z^{k+1})$ satisfying equations (12a), (12b) and

$$W(p, q, U^{k+1}) < W(p, q, U^k) \text{ if } W(p, q, U^k) > 0 \quad (15)$$

where $U_i^{k+1} = u_i\left(x_0^{k+1}, x_i^{k+1}, z^{k+1}, d\right), U^{k+1} = (U_1^{k+1}, \dots, U_n^{k+1})$. $\Delta_i^{k+1} = [e_i(p_i, x_0^{k+1}, z^{k+1}, U_i^{k+1}) - e_i(p_i, x_0^{k+1}, z^{k+1}, U_i^t)]$, $i \in N$, and $M^{k+1} = \max_i \{\Delta_i^{k+1}: i \in N\}$. Then, let $k = k + 1$ and go to step S2.

Scheme S^E differs from S in two ways. First, $W(p, q, U^k) = 0$ has been added in step S^E3a. From Proposition 1, this corresponds to Pareto efficiency. Second, when $W(p, q, U^k) > 0$, equation (15) has been added to the iterative scheme. Equation (15) states that the bargaining process must reduce the distance to the Pareto utility frontier across iterations from $k$ to $k + 1$

implying that the iterative process necessarily moves toward the Pareto utility frontier. Upon convergence, from $S^E$3a, this leads to an allocation on the Pareto utility frontier. Building on Proposition 2, we have the following result (see the proof in Appendix A).

Proposition 3: Upon convergence, the bargaining scheme $S^E$ identifies an efficient bargaining agreement $(x^{k^*}, y^{k^*}, z^{k^*})$ that corresponds to a unique point on the Pareto utility frontier.

Proposition 3 states that the bargaining process given in $S^E$ converges to a unique point on the Pareto utility frontier. The bargaining schemes S and $S^E$ have some nice properties. First, they involve simple iterations that may reflect the steps taken during an actual bargaining session. Second, upon convergence, scheme $S^E$ finds a bargaining agreement for an allocation that is Pareto efficient and where all individuals are equally willing to face a bargaining failure. Third, the bargaining process $S^E$ identifies a unique point on the Pareto utility frontier. This unique point depends on the threat points, thus stressing the importance of bargaining failure.

Note that the identification of a unique point on the Pareto utility frontier was obtained from an intra-household bargaining process. It did not require the specification of a household preference function. Yet, a household preference function can always be found to rationalize observed behavior. For example, if the Pareto utility frontier is concave, then there exists a hyperplane tangent to the utility frontier at some evaluation point. In this case, the slopes of this hyperplane can be treated as Bergsonian weights in a household utility function. But changing these Bergsonian weights can rationalize any point of the utility frontier, meaning that a household preference function does not really help identifying the factors affecting distribution issues. In contrast, our bargaining approach and its reliance on threat points do provide the additional information we can use in the investigation of income and welfare distribution within the household.

Is there a simple representation of the outcome of the bargaining process $S^E$? The answer is given in the following proposition (see the proof in Appendix A).

Proposition 4. Consider the following optimization problem

$$\underset{x_0, y, z, U}{\text{Max}} \{ \prod_{i \in N} [e_i(p_i, x_0, z, U_i) - e_i(p_i, x_0, z_0, U_i^t)]:$$
$$\textstyle\sum_{i \in N} e_i(p_i, x_0, z, U_i) + p_0 x_0 \leq qy,$$
$$e_i(p_i, x_0, z, U_i) \geq e_i(p_i, x_0, z_0, U_i^t), i \in N; x_0 \in X_0, (y, z) \in F\}, \tag{16}$$

which has for solution $(x_0^e, y^e, z^e, U^e)$. The allocation $(x_0^e, y^e, z^e, U^e)$ is an efficient bargaining agreement.

Proposition 4 is an extension of Nash bargaining under ordinal preferences (Nash, 1950). As such, the term $\prod_{i \in N}[e_i(p_i, x_0, z, U_i) - e_i(p_i, x_0, z, U_i^t)]$ in (16) is a generalized "Nash product". The maximization problem in (16) subject to two sets of constraints: the budget constraint $\sum_{i \in N} e_i(p_i, x_0, z, U_i) + p_0 x_0 \leq qy$, and the incentive compatibility constraint $e_i(p_i, x_0, z, U_i) \geq e_i(p_i, x_0, z, U_i^t), i \in N$. When combined with Proposition 3, Proposition 4 states that the constrained maximization of the generalized Nash product in problem (16) picks a unique point on the Pareto frontier that is a bargaining agreement as identified in scheme $S^E$.

While Proposition 4 identifies scenarios where bargaining leads to efficient allocations (under bargaining scheme $S^E$), we also considered allocations associated with bargaining agreements that may be inefficient. In general, the bargaining scheme S can generate bargaining outcomes that are inefficient. Clearly, inefficiency could arise only if the individuals involved in the bargaining process fail to identify the benefits from efficiency gains. This can happen if the individuals have difficulties in identifying these benefits, for example due to information cost and/or cognitive limitations bounding individual rationality.

In this context, it is useful to examine what are the efficiency cost of these agreements. Consider a bargaining agreement $(x^a, y^a, z^a)$ associated with scheme S. How does it compare with an efficient bargaining agreement $(x^e, y^e, z^e)$ associated with scheme $S^E$? Let $U^a = \{(U_1^a, \dots, U_n^a) \colon U_i^a = u_i(x_0^a, x_i^a, z^a), i \in N\}$. Let $U^e = \{(U_1^e, \dots, U_n^e) \colon U_i^e = u_i(x_0^e, x_i^e, z^e), i \in N\}$. From Proposition 1, note that $W(p, q, U^e) = 0$ under efficiency, while $W(p, q, U^a) \geq 0$. Then the efficiency cost of a bargaining agreement $(x^a, y^a, z^a)$ can be measured as

$$\Delta W^a = W(p, q, U^a) \geq 0. \tag{17}$$

Equation (17) gives as $\Delta W^a$ as a welfare measure of the distance between the bargaining agreement $(x^a, y^a, z^a)$ and the Pareto utility frontier. When bargaining leads to efficient outcomes then $\Delta W^a = 0$ (from Proposition 1). But when bargaining agreements lead to allocations that fall short of efficiency, then $\Delta W^a > 0$. In this case, equation (17) evaluates the welfare cost of imperfect bargaining.

Note that there are many ways for a household to be inefficient. From Corollary 1, inefficiency can arise when $x_0$ does not satisfy (5), when $y$ does not satisfy (6), when $z$ does not satisfy (7) and/or when the household budget constraint (1) is not binding. Given this complexity, is there a simple way to represent inefficient agreements? As indicated in Oprea (2020), the answer is affirmative. Consider the case where imperfections in household bargaining can be represented by a bargaining cost $c$. This bargaining cost can provide a rational for inefficient agreements: the household would stop bargaining when $\Delta W^a < c$, i.e., when the benefit of bargaining is less than its cost. This suggests introducing bargaining cost in the evaluation of bargaining agreements. This can be done by modifying Proposition 4 as stated next. The proof is similar to Proposition 4 and is omitted.

Proposition 5:

a/ if $c \geq W(p, q, d, U^t)$, there is no bargaining agreement, and the household allocation is the threat point $(x_0^t, y^t, z^t, U^t)$. (18a)

b/ if $0 \leq c < W(p, q, d, U^t)$, consider the optimization problem

$$\underset{x_0,y,z,U}{\text{Max}} \{ \prod_{i \in N} [e_i(p_i, x_0, z, U_i) - e_i(p_i, x_0, z, U_i^t)]:$$

$$\sum_{i \in N} e_i(p_i, x_0, z, U_i) + p_0 x_0 \leq qy - c,$$

$$e_i(p_i, x_0, z, U_i) \geq e_i(p_i, x_0, z, U_i^t), i \in N; x_0 \in X_0, (y, z) \in F; \}, \quad (18b)$$

which has for solution $\left(x_0^b, y^b, z^b, U^b\right)$. Then, the allocation $\left(x_0^b, y^b, z^b, U^b\right)$ is a bargaining agreement which is efficient if $c = 0$ but inefficient if $c > 0$.

Proposition 5 proposes a simple representation of inefficient bargaining agreements, conditional on $c$. Equation (18a) states that household bargaining fails, generating the threat allocation $(x_0^t, y^t, z^t, U^t)$, when bargaining cost is larger than bargaining benefit: $c \geq W(p, q, U^t)$. And equation (18b) generalizes (16) by introducing cost $c$ in the budget constraint. Like (16), the solution to (18b) is a bargaining agreement (see the proof in Proposition 4). When $c = 0$, (18b) reduces to (16), which identifies an efficient bargaining agreement (from Proposition 4). But when $0 \leq c < W(p, q, U^t)$, (18b) provides useful information on inefficient bargaining agreements. In general, the solution $\left(x_0^b, y^b, z^b, U^b\right)$ to (18b) varies with $c$. Evaluating (18b) for different values of $c \geq 0$ generates a set of bargaining agreements going from an efficient agreement (when $c = 0$), to inefficient agreements (when $0 < c < W(p, q, U^t)$ and to no agreement (when $c \geq W(p, q, U^t)$)). In this context, following Oprea (2020), $c$ can be interpreted as a measure of cost that reflects both the degree of complexity and the degree of inefficiency: the agreements represented by (18b) become increasing inefficient as $c$ increases from 0 to $W(p, q, U^t)$.

This is illustrated in Figure 1 in the case where $n = 2$. Figure 1 shows the range of utilities $(U_1, U_2)$ that can be obtained under alternative allocations. The Pareto utility frontier is given by the line AEC, representing the set of utilities $(U_1, U_2)$ where $W(p, q, U_1, U_2) = 0$. The set of utilities obtained under bargaining agreements represented by (18a-18b) is given by the line TBE, that we label bargaining or contract curve. This line starts at point T, representing the threat points $(U_1^t, U_2^t)$ obtained when bargaining fails: when $c \geq W(p, q, U^t)$. As $c$ decreases from $W(p, q, U^t)$ toward 0, the line TBE represents inefficient bargaining agreements as one moves up toward the Pareto utility frontier. When $c = 0$, the line TBE crosses the Pareto utility AEC at point E, identifying point E as an efficient agreement. As illustrated in Figure 1, any move up the line TBE corresponds to a Pareto improvement as it would make both individuals better off. Point E is unique: it is the only point $(U_1, U_2)$ that is both efficient and a bargaining agreement. This illustrates a key result from Propositions 3 and 4. The bargaining scheme $S^E$ generates a unique point on the Pareto utility frontier. This is one of our important results: combining bargaining and efficiency settles the issue of how welfare is distributed within the household. But scheme S opens the possibility that bargaining does not always lead to efficient outcomes. Figure 1 shows that any point along the line TBE is a possible bargaining agreement. Point B represents one of those agreements. Point B is inefficient (it is below the Pareto utility frontier AEC) and the inefficiency is measured by the distance between points B and E.

[Figure 1 about here]

## 6. Threat Strategies

What determines the threat strategies $(x_{0i}^t, x_i^t, y_i^t, z_i^t), i \in N$? In this section, we discuss alternative approaches to the investigation of threat strategies. The first approach involves a situation where the threats imply breaking up the households into $n$ parts, each household member managing his/her resources independently. In this case, the threat points would correspond to the exit options available to each individual. An example would be the case of divorce applied to a married couple, which would represent an *extrema ratio* solution (Manser and Brown, 1980; McElroy and Horney, 1981). The threat to break-up the sentimental relation is probably the most powerful among the possible threats. It corresponds to the level of utility, net of transaction or emotional costs, that would be achieved if living alone. Compared to a threat based on a non-cooperating decision, it has the practical advantage of defining the maximum size of the negotiation set. In this context, the threat points $(x_{0i}^t, x_i^t, y_i^t, z_i^t) \in X_0 \times X_i \times F$ would be obtained from applying the analysis presented above to each individual, $i \in N$.[9]

The second approach would consist in associating the threats to non-cooperative behavior within the household (Lundberg and Pollak, 1993, 1996; Ulph, 1988; Woolley, 1988). In this case, the threat points are internal to the marriage, not external as in situations that consider divorce as a credible threat (Lundberg and Pollak, 1993, 1996). These disagreement situations would be identified as the outcomes of a non-cooperative game played by the $n$ household members. For example, such outcomes would be equilibrium points in non-cooperative, voluntary contribution Nash equilibrium (Nash, 1951; Rosen, 1965). As noted by Lundberg and Pollak (1996:147) "noncooperative marriage in which the spouses receive some benefits due to joint consumption of public goods may be a more plausible threat in day-to-day marital bargaining." Deciding to live non-cooperatively in separate spheres within the same house is also a strong threat especially if sustained through time. It is probably a strategy that is not commonly adopted in ordinary conflicts. By reinterpreting an inefficient bargaining agreement as a non-cooperative threat with respect to a movement towards the efficient agreement, in the sense that the couple decides not to cooperate to do an extra effort to move closer to the Pareto frontier, as we do in our study, we are also adopting a milder version of the separate sphere noncooperative threat.

A third approach would involve the role of social rules and their effects on the resolution of conflicts within the household. Such social rules can guide and constrain socio-economic behavior of household members. As discussed in Ellickson (2008), these rules can be formal (e.g., explicit contracts) as well as informal (e.g., implicit contracts developed and managed within the household). Social rules play a role when conflicts arise among household members. Conflict resolution mechanisms can help regulate bargaining failure and affect the threat points used in our analysis. This includes the role of the Courts in settling disputes (including divorce settlements). These arguments stress that the determinants of threat points are complex and vary with the socio-political environment.

[9] Interestingly, Boto-García and Perali (2020) find that this exit option is more frequent than one may expect. Around 44% of their sample has seriously considered to end the relationship with their current partner in the past. The intention to break-up is more frequent among those who score low in marital locus of control, males, low-income earners, individuals with university studies and couples without children.

## 7. Empirical Application of the Household Bargaining Model

In this section we take Propositions 3 and 5 to the data. Proposition 3 identifies efficient bargaining outcomes employing the efficient bargaining scheme $S^E$ based on an *iterative* process to reach an agreement.[10] Proposition 5 generates a set of bargaining agreements using the notion of bargaining cost associated with the individual willingness-to-pay to avoid the breakdown position. Unlike Proposition 3, Proposition 5 involves an *optimization* process.

### 7.1. Data, Individual Preference Structure and Household Technology

The analysis is carried out using a nationwide survey on socioeconomic characteristics of Italian rural households undertaken in 1995 by the Italian Institute for Agricultural Markets (ISMEA). The farm-household survey combines information about household and farm characteristics, farm production and profits, stylized time use, off-farm labor income, governmental and intra-household transfers, consumption, and information on the degree of autonomy in decision making by household members. A relevant feature of our data is information about the private consumption of assignable goods, such as clothing for women and men, which is sufficient to identify the bargaining power as defined in equation (13), a concept analogous to the sharing rule governing the intra-household allocation of resources (Chiappori 1988, 1992; Chiappori and Ekeland, 2009).

The empirical implementation of the household bargaining process is based on a collective family enterprise model (Matteazzi, Menon and Perali, 2017) described in Appendix B. We next investigate how agreements are settled and explore the link between the bargaining position and the distribution of income within the household as discussed in Section 4. We then study the determinants of the occurrence of a bargaining agreement, an efficient allocation, and an efficient bargaining agreement. We explore how families differ in the speed in reaching an agreement, as measured by the number of iterations employed to find an arrangement, and in the cost of inefficiency, as measured by the distance between the settled inefficient agreements and the Pareto frontier.

### 7.2. Implementation of the Bargaining Process

We apply the bargaining algorithm of Proposition 3 assuming that two individuals ($n = 2$), the wife ($i = 1$) and the husband ($i = 2$), bargain over household resources. For each spouse the threat point is the level of private expenditure of a person living alone corresponding to her purchasing power when bargaining fails. To investigate how the threat point influences the

[10] We present only the bargaining scheme $S^E$ because it encompasses the bargaining scheme S.

bargaining process and the final location on the utility Pareto frontier, we replicate the bargaining iterative process for different threat points. The iteration process in $S^E$ is mimicked using a stochastic simulation that generates 1,000,000 feasible allocations. The detailed description of the process is reported in Appendix C.

Figures 2 provides a graphical representation of the $S^E$ scheme. The figure traces the feasible negotiation set where spouses have a utility level equal to or greater than the individual utility associated with the threat point. For the average family, at the breakdown position the spouses have a similar purchasing power: 1,622€ for the wife and 1,533€ for the husband. The grey line, starting from the threat point and ending on the Pareto frontier, is the set of bargaining agreements defining the bargaining curve. It corresponds to the TBE blue line in Figure 1. Along the bargaining curve, spouses have the same willingness-to-pay to avoid a bargaining failure, which is measured as the difference between the randomly generated individual private expenditure and individual purchasing power at the threat point. The point on the Pareto frontier corresponding to the allocation randomly generated at iteration $k$ = 136,526 is the unique efficient agreement. Only 1.4% of randomly generated allocations (14,142 out of 1,000,000) produce a bargaining agreement. The set of efficient bargaining agreements, where the value of the household maximized net income ($W$) is very close to zero, represents the 0.2% of the cases, suggesting that there exists significant inefficiency within the family. Efficient allocations are desirable outcomes, but, more realistically, family life tends to accept sufficient levels of satisfaction even though inefficient.

[Figure 2 about here]

Proposition 5 shows how to identify the set of efficient and inefficient bargaining agreements conditional on bargaining cost $c$.[11] We solve the optimization model of equations (18a) and (18b) for different values of $c$. Looking at Table 1, when $c$ = 0 the spouses are on the Pareto frontier with a utility level equal to 7.7 for the wife and 10.9 for the husband corresponding to the efficient

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[11] The empirical implementation of Proposition 5 involves solving the following set of equations

$$\frac{1}{e_1\left(p_1,x_0,z,u_1\left(x_0,x_1,z\right)\right)-e_1(p_1,x_0,z,U_1^t)}=\lambda,\quad \frac{1}{e_2\left(p_2,x_0,z,u_2\left(x_0,x_2,z\right)\right)-e_2(p_2,x_0,z,U_2^t)}=\lambda,$$

$$e_1\big(p_1,x_0,z,u_1(x_0,x_1,z)\big)+e_2\big(p_2,x_0,z,u_2(x_0,x_2,z)\big)=y-c,$$

where $y$ is household income, $c$ is bargaining cost, and $\lambda$ is the Lagrange multiplier of the family budget constraints. The specification of the individual expenditure functions is in Appendix B.

allocation reached at iteration $k$ = 136,526 (Table C1 in Appendix C). When the bargaining cost is set to 2,666€, corresponding to the welfare loss associated with the threat allocation, the utility level is 6.4 for the wife and 9.6 for the husband. For values of $c \in (0, 2666)$, the optimal solutions of the program correspond to bargaining agreements but inefficient (grey curve in Figure 2).

[Table 1 about here]

### 7.3. Threat Point, Bargaining and Intra-Household Inequality

This section shows 1/ how the bargaining process $S^E$ generates Pareto-improving redistributions of resources, and 2/ how the threat point affects both the location of the bargaining agreement on the Pareto frontier and the intra-household inequality.

In Panel A of Figure 3, at the threat point ($\sigma_2 = 0.486$ and $\sigma_1 = 1 - \sigma_2 = 0.514$) the husband ($i$ = 2) and wife ($i$ = 1) have similar expenditure shares. Panel B shows the case of a higher expenditure share of the wife ($\sigma_1 > \sigma_2$). At the threat point, the private expenditure of the wife is 2,085€, while the private expenditure of the husband is 657€, with an expenditure share $\sigma_2$ equal to 0.24 ($\sigma_1 = 0.76$), suggesting that the husband faces a relatively more damaging threat than his wife. In Panel C, the husband is favored at the threat point. His purchasing power is 1,751€, against 927€ for his wife, with an expenditure share $\sigma_2$ equal to 0.65. In all three panels, moving upward along the bargaining curve from the threat point towards the Pareto frontier, equity improves. When the couple achieves a bargaining agreement, the intra-household allocation of resources becomes more equal, indicating that the bargaining process is Pareto and equity improving.

[Figure 3 about here]

Figure 3 also shows that the degree of equity of process $S^E$ depends on the threat point. If at the threat point resources are (almost) equally allocated within the couple, then equality is maintained at the final location on the Pareto utility (Panel A).[12] On the other hand, Panel B and C show that if resources are unequally allocated between spouses at the threat point, then the final location on the Pareto utility frontier remains unequal, though less so. This is evidence that the bargaining process may be unable to fully adjust for equity even when efficiency is reached. At the efficient bargaining agreement, the allocation of economic resources is more equal than the one at the threat point, suggesting that the bargaining process is equity improving as measured by the Shannon entropy index $-\sum_{i=1}^{2} \sigma_i \ln \sigma_i$ developed by Chavas et al. (2018) and presented in

[12] In Appendix C, Figure C1 shows the bargaining process $S^E$ starting at an egalitarian threat point.

Panel D. The inequality index takes values in the range $(0,1)$ and has an inverted U-shape. For egalitarian allocations ($\sigma_i = 0.5$, $i = 1,2$), the Shannon index takes its highest value of 0.693. The lower the index, the higher the degree of inequality within the couple. As shown in Panel D, at the Pareto frontier, the value of the Shannon index is equal to or higher than its value at the threat point and closer to 0.693, implying that the negotiation process improves equity within the couple. When individuals have similar purchasing powers at the threat point (Panel A), the Shannon index is close to its maximum value of 0.639 and remains unchanged at the efficient bargaining agreement. The size of the equity-improving effect of the bargaining process is larger when at the threat points are in favor of one spouse (Panel B and C).

## 8. Determinants of Bargaining Outcomes

Here we address the following questions: a) What are the factors affecting agreeableness? b) How difficult is it to reach an agreement? and c) How does the cost of inefficiency vary across households?

The analysis is carried out using data generated by replicating the efficient bargaining scheme $S^E$. For each family, we randomly generate $k = 2{,}000$ feasible allocations.[13] At each iteration the family can either fail to reach an agreement, be on the contract curve, or move toward the contract curve up to an efficient point on the Pareto utility frontier. Within the selected range of $k$ iterations, in our sample 93% of families reach at least a bargaining agreement, 40% reaches an efficient allocation and only 9% reaches a bargaining agreement that is efficient.

### 8.1 Factors Affecting Agreeableness

We study the determinants of the likelihood of occurrence of the bargaining outcomes by estimating three logit models: 1) the probability of reaching a bargaining agreement (93%), 2) the probability of reaching Pareto efficiency (40%), and 3) the probability of simultaneously reaching a bargaining agreement and an efficient allocation (9%). The regressors include a set of variables measuring the intra-household bargaining power of the husband. The husband control over resources is captured by a  dummy equal to one if at the threat point his expenditure share is larger than 0.5. The husbands outside options when bargaining fails are proxied by the husband-wife

[13] This choice is based on the trade-off between computational burden and robustness. At the ceiling of about 2,000 allocations the number of families reaching an agreement remains almost unchanged and those reaching an efficient bargaining agreement slightly increases (Table D1 in Appendix D).

wage ratio and a dummy equal to one if the husband inherited the farm activity. We also construct an ordinal index describing who is the principal decision maker when it comes to decisions related to business, the family, job opportunities, and household finance. For each of these decisional spheres, we define a variable taking value 0 if the wife is the sole decision-maker, 1 if the husband and the wife decide together, and 2 if the husband is the sole decision-maker. Summing over the four domains, we associate a score in the [0,8] range to the husband decision-making power index. At 0 the husband does not have power in any of the four domains, while at 8 the husband acts as "a dictator." Other variables are the educational dummy equal to one if both spouses have at least a lower secondary education, the age classes of the husband, the farmers risk propensity, macro-regions (North, South, Center), and the number of wage earners in the family. Descriptive statistics are in Table D2 of Appendix D.

Table 2 shows the predicted probabilities and average marginal effects of the three logit models. A household with a threat allocation more favorable to the husband has a probability of reaching a bargaining agreement 8% points lower than a family where the wife has better outside options than her spouse (column 1). This suggests that households where the husband has the most to lose from a failed negotiation are less likely to come to an agreement revealing that men may have a lower propensity to make concessions. The allocation at the threat point is not significant for the other two outcomes (column 2 and 3). On the other hand, an increase in the decision-making power of the husband results in a reduction of the probability of reaching an efficient allocation (column 2). In a household where the husband has little decision-making power, the predicted probability of attaining efficiency is 55%, against 31% when he acts as a "dictator". These results suggest that inefficiency tends to be lower when wives are more involved in the household decision process. An increase of one standard deviation in the wage ratio results in a 2.5% points reduction in the probability of achieving an efficient bargaining agreement (column 3).

[Table 2 about here]

### 8.2 Difficulties in Reaching an Agreement

Descriptive statistics shows that families are highly heterogeneous with respect to the speed with which they reach an agreement (Table D2 in Appendix D). Reaching an efficient (inefficient) outcome may take a short or long negotiation process. We refer to families where the outcome is reached within the first 50 iterations as *fast* families; in contrast, *slow* families attain an agreement after the 50-th iteration.[14]

Table 3 shows the estimation results of a multinomial logit model analyzing the determinants of the speed of the bargaining process.[15] In general, more favorable outside options for the husband are associated with a higher probability of a bargaining failure. Focusing on efficient allocations, *fast* and *slow* families (column 5 and 6) have similar bargaining behavior, except for the role of the decision-making power of the husband that increases the probability of failure to achieve efficiency by 3.2% points (column 4). It decreases by a similar amount the probability of being a *slow* family (column 5), but it does not significantly affect the probability of being a *fast* family. The predicted probability of being a *fast* family is 16% when the husband has no decision-making power, and 20% when he acts as a dictator. On the other hand, the probability of failing to reach efficiency is 43% when the husband has no decision-making power and 69% when he acts as a dictator. These probabilities are 41% and 11% for *slow* families.

[Table 3 about here]

**8.3 Cost of Inefficiency Across Households**

We examine the determinants of the cost associated with inefficient outcomes, measured as the difference between total household income $\pi$ and the sum of the spouses' private expenditures, $W = \pi - \sum_{i=1}^{2} e_i\big(p_i, x_0, z, u_i(x_i, x_0, z)\big)$, by calculating the individual level of private expenditure $e_i$ at each randomly generated allocation. To account for household heterogeneity, we estimate a linear mixed-effect model with random intercepts.

The husband decision-making power has a positive and significant effect because it raises the cost of imperfect bargaining (Table 4). The estimated cost for a family whose husband has no decision power amounts to 3,067€, while it increases up to 5,395€ for a family whose husband acts as a "dictator". This finding is strictly related to the evidence in Table 3, where the index for the husband's decision-making power negatively affects the probability of reaching efficiency.

[Table 4 about here]

[14] We cannot undertake this analysis for the efficient bargaining agreements because of the small size of this subsample.
[15] The Hausman-McFadden test does not reject the null that the IIA assumption is not violated.

## 9. Conclusions

This study revisits the Nash-Harsanyi bargaining model making it closer to the day-to-day functioning of a family and a more realistic representation of the decision process. Our approach recognizes that negotiation is costly and often difficult explaining why family members may accept Pareto inefficient bargaining agreements that make the family disputants sufficiently happy. Our focus is not only on the efficient outcomes of the decision process, but also on the bargaining process as it evolves inside the negotiation set. Our evolutionary bargaining approach identifies who is willing to make a concession depending on the perceived cost of bargaining failure. We extend the Nash-Harsanyi cardinal representation to ordinal preferences; and we consider agreements that may not be efficient.

We illustrate the usefulness of our theory in an application that reveals the many interesting and meaningful traits of a previously unexplored empirical content of the bargaining model. The outcomes of the bargaining schemes show that most of the agreements are not efficient, lending empirical support to Simon's hypothesis that rational individuals can be sufficiently satisfied also at inefficient but less conflictual positions on the contract curve. The bargaining process reduces intra-household inequalities but does not necessarily lead to an equal allocation when efficiency is reached. The higher the initial level of utility at the threat point, the higher the final utility at the agreement point.

The study of the determinants of the bargaining agreements is also very informative. In general, a household with a threat allocation more favorable to the husband has a lower probability of reaching a bargaining agreement than a family where the wife has better outside options. This suggests that households where husbands have the most to lose from a failed negotiation are less likely to come to an agreement. In a household where the husband holds little decision-making power, the probability of attaining efficiency is significantly higher as compared to a situation where the husband holds most of the power. Inefficiency tends to be lower when wives are more involved in the household decision process. The odd of attaining an efficient bargaining agreement is higher as the wife's earnings increase.

Families differ substantially also in the speed in reaching an agreement. Families that are *fast* in reaching a bargaining agreement are relatively older and less risk averse. On the other hand, *fast* and *slow* families appear to have similar bargaining behavior when engaged in reaching efficient agreements, except for the role of the bargaining power of the husband. In *slow* families, preponderant husbands are less influential. Welfare costs associated with bargaining are in general positively related with the bargaining power of the husband, his age and risk propensity and the education level of the wife.

While we have presented a general conceptual approach to intra-household bargaining, there is a need to extend its applications to specific economic issues. This includes studies of labor supply choices, the harmonization of work and family duties, task specialization within the household, fertility choices, and antisocial behavior such as the case of domestic violence. It would also be useful to explore the linkages between intra-household behavior and economic policy. Addressing these issues seems to be good directions for future research.

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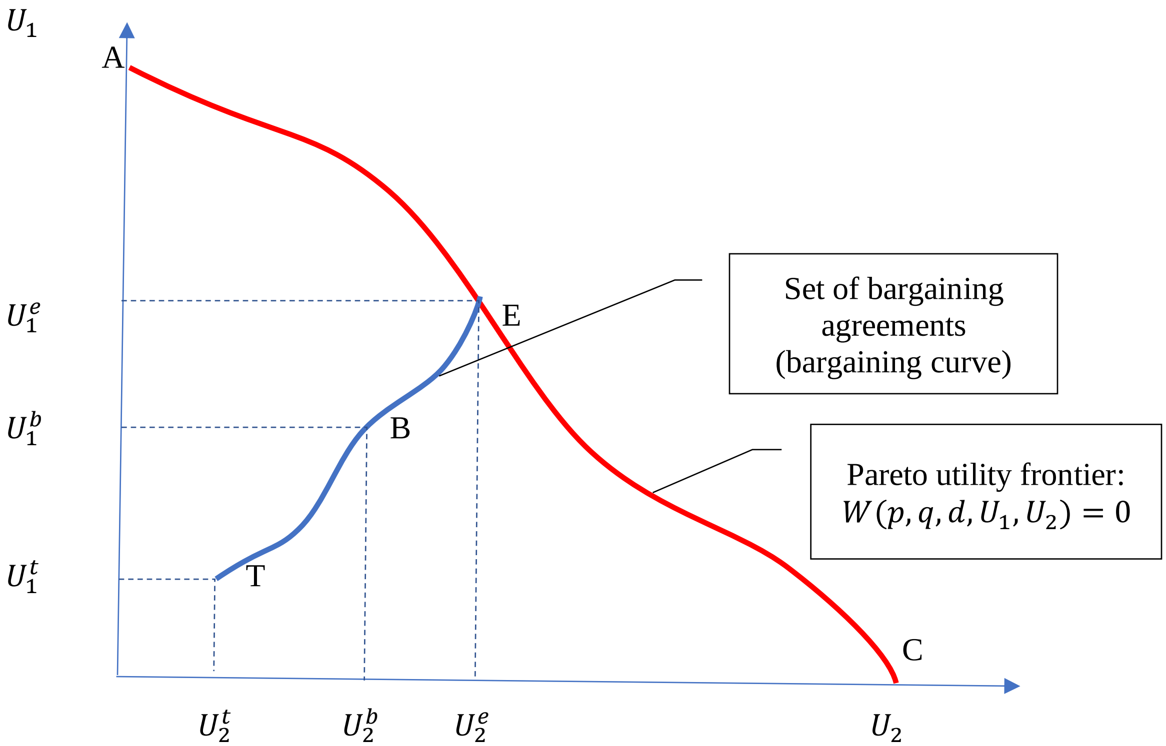
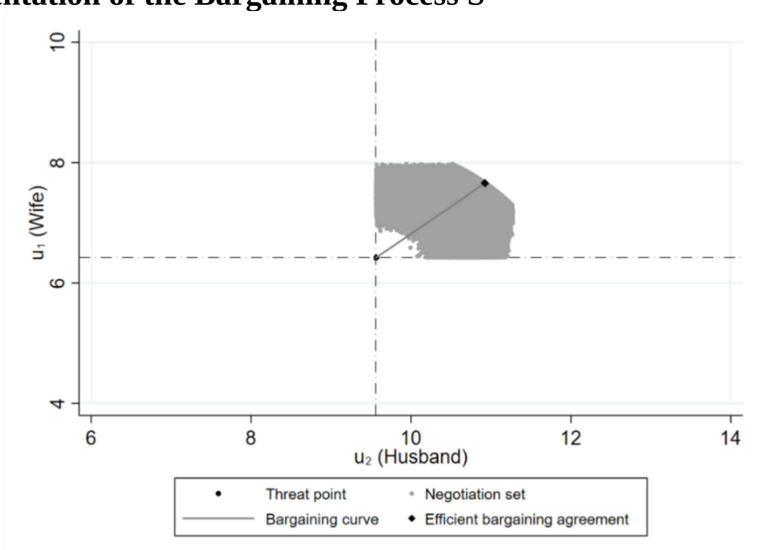
## Figures

### Figure 1: Efficiency and Bargaining



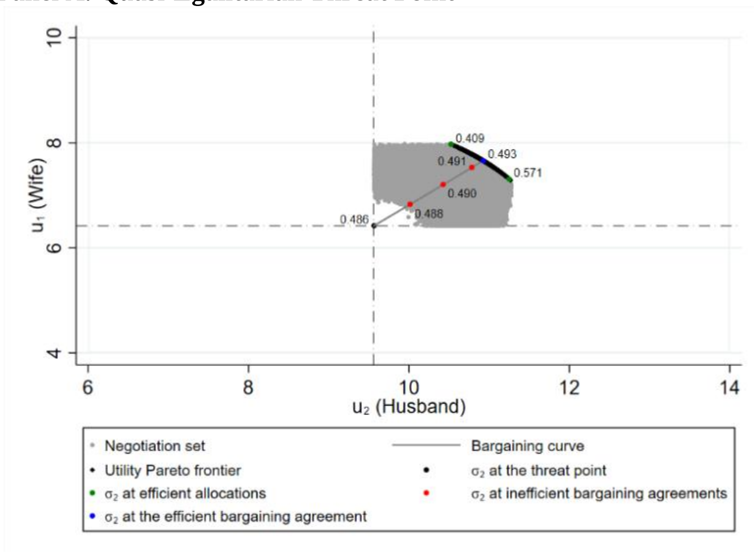

### Figure 2: Representation of the Bargaining Process $S^E$



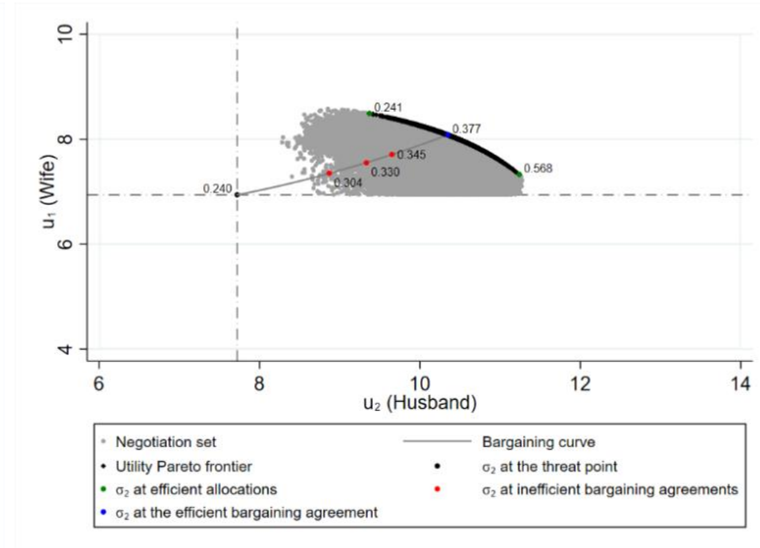

Notes: Individual utility at the threat point: $u_1^t = 6.4$, $u_2^t = 9.6$, individual utility on the Pareto frontier: $u_1^E = 7.7$, $u_2^E = 10.9$.

## Figure 3: Threat Point, Bargaining and Intra-Household Inequality

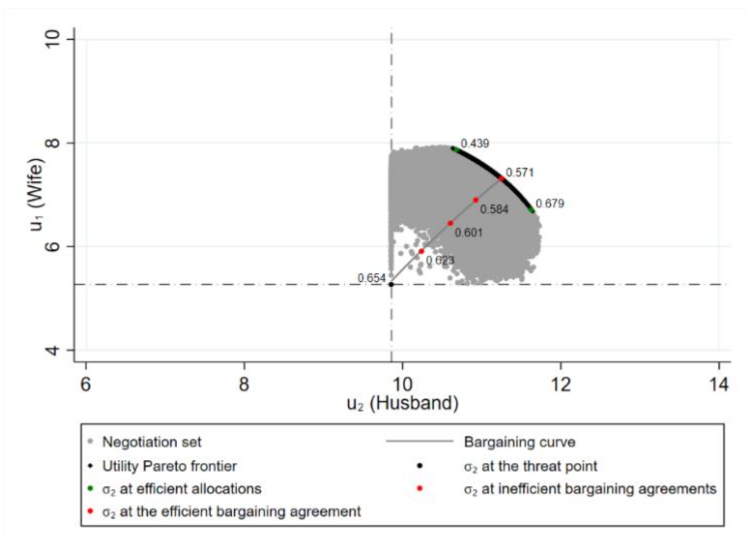
**Panel A. Quasi-Egalitarian Threat Point**




**Panel B. Threat Point More Favorable to the Wife**




**Panel C. Threat Point More Favorable to the Husband**




**Panel D. Intra-Household Inequality**

| | Threat point | | Pareto frontier | |
|---|---|---|---|---|
| | $\sigma_2$ | Shannon index | $\sigma_2$ | Shannon index |
| Panel A | 0.486 | 0.693 | 0.493 | 0.693 |
| Panel B | 0.240 | 0.551 | 0.377 | 0.661 |
| Panel C | 0.654 | 0.645 | 0.571 | 0.683 |

Note: $\sigma_2$ is the husband resource share. Shannon index $= -\sum_{i=1}^{2} \sigma_i \ln\sigma_i$ takes values in the range $(0,1)$ and has an inverted U-shape relationship. When resources are equally allocated between the spouses ($\sigma_1 = \sigma_2 = 0.5$), the associated Shannon index is 0.693.

# Tables

## Table 1: Bargaining Costs and Individual Utility Levels

| Bargaining cost – *c* | $U_1$ | $U_2$ |
|---|---|---|
| 0 | 7.7 | 10.9 |
| 298.3 | 7.6 | 10.8 |
| 565.4 | 7.5 | 10.7 |
| 1589.6 | 7.0 | 10.2 |
| 1948.4 | 6.8 | 10.0 |
| 2666.1 | 6.4 | 9.6 |

Notes: Bargaining costs are in euro per month.

## Table 2: Probabilities and Average Marginal Effects of Logit Model (726 Families)

| | Outcome | | |
|---|---|---|---|
| | Bargaining agreement | Efficient allocation | Efficient bargaining agreement |
| | (1) | (2) | (3) |
| **Predicted probability** | 0.923*** | 0.401*** | 0.079*** |
| | (0.010) | (0.037) | (0.010) |
| **Average marginal effects** | | | |
| Husband's share at the threat point > 0.5 | −0.073** | 0.033 | 0.045 |
| | (0.034) | (0.046) | (0.033) |
| Husband's market wage/wife's market wage | −0.120 | 0.131 | −0.575** |
| | (0.239) | (0.423) | (0.242) |
| Husband's decision-making power | 0.006 | −0.030** | −0.001 |
| | (0.007) | (0.012) | (0.007) |
| Husband inherited the farm | 0.017 | 0.046 | −0.002 |
| | (0.022) | (0.037) | (0.021) |
| Medium-highly educated husband | 0.027 | −0.058 | −0.024 |
| | (0.024) | (0.042) | (0.023) |
| Medium-highly educated wife | −0.005 | −0.095 | −0.035 |
| | (0.039) | (0.060) | (0.028) |
| Husband aged 35-59 | 0.078* | −0.045 | −0.059 |
| | (0.044) | (0.062) | (0.044) |
| Husband aged 60 and over | 0.057** | 0.007 | −0.041 |
| | (0.026) | (0.076) | (0.031) |
| Medium propensity to take risks | 0.022 | −0.063 | 0.005 |
| | (0.022) | (0.042) | (0.026) |
| High propensity to take risks | 0.014 | −0.072* | 0.003 |
| | (0.022) | (0.041) | (0.024) |
| North | 0.037 | −0.297*** | −0.084*** |
| | (0.026) | (0.043) | (0.024) |
| South | 0.012 | −0.114*** | 0.003 |
| | (0.027) | (0.043) | (0.026) |
| Number of wage earners | 0.024** | −0.068*** | 0.001 |
| | (0.011) | (0.017) | (0.009) |

Notes: Reference category for husband’s age: younger than 35. Reference category for risk behavior: low propensity to take risks. Reference category for macro-regions: Center. Standard errors are in parenthesis. *** p<0.01, ** p<0.05, * p<0.10.

**Table 3. Probabilities and Average Marginal Effects of Multinomial Logit Models**

| | Bargaining agreement | | | Efficient allocation | | |
|---|---|---|---|---|---|---|
| | Failed bargaining | Slow Families | Fast families | Failed efficiency | Slow families | Fast families |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| **Predicted probabilities** | 0.077*** | 0.521*** | 0.402*** | 0.599*** | 0.205*** | 0.196*** |
| | (0.010) | (0.018) | (0.018) | (0.017) | (0.015) | (0.014) |
| **Average marginal effects** | | | | | | |
| Husband's share at the threat point > 0.5 | 0.074** | −0.054 | −0.020 | −0.035 | −0.041 | 0.076* |
| | (0.034) | (0.050) | (0.047) | (0.047) | (0.037) | (0.043) |
| Husband's market wage/wife's market wage | 0.117 | −0.889* | 0.772* | −0.152 | 0.335 | −0.183 |
| | (0.239) | (0.457) | (0.450) | (0.423) | (0.359) | (0.345) |
| Husband's decision-making power | −0.006 | −0.008 | 0.014 | 0.032*** | −0.035*** | 0.003 |
| | (0.007) | (0.013) | (0.013) | (0.012) | (0.011) | (0.010) |
| Husband inherited the farm activity | −0.017 | 0.072* | −0.055 | −0.047 | 0.054* | −0.007 |
| | (0.022) | (0.040) | (0.039) | (0.037) | (0.031) | (0.031) |
| Medium-highly educated husband | −0.028 | 0.052 | −0.025 | 0.059 | −0.061* | 0.001 |
| | (0.024) | (0.044) | (0.043) | (0.042) | (0.036) | (0.034) |
| Medium-highly educated wife | 0.006 | 0.026 | −0.033 | 0.093 | −0.035 | −0.058 |
| | (0.040) | (0.066) | (0.065) | (0.061) | (0.052) | (0.046) |
| Husband aged 35-59 | −0.078* | −0.051 | 0.129** | 0.043 | 0.025 | −0.068 |
| | (0.044) | (0.066) | (0.062) | (0.063) | (0.054) | (0.055) |
| Husband aged 60 and over | −0.057** | 0.000 | 0.057 | −0.015 | 0.046 | −0.030 |
| | (0.026) | (0.083) | (0.083) | (0.078) | (0.074) | (0.057) |
| Medium propensity to take risks | −0.022 | −0.072 | 0.094** | 0.062 | −0.015 | −0.048 |
| | (0.022) | (0.045) | (0.045) | (0.042) | (0.036) | (0.033) |
| High propensity to take risks | −0.014 | −0.026 | 0.040 | 0.071* | −0.011 | −0.060* |
| | (0.022) | (0.045) | (0.045) | (0.041) | (0.035) | (0.032) |
| North | −0.037 | −0.069 | 0.106** | 0.295*** | −0.147*** | −0.148*** |
| | (0.026) | (0.052) | (0.051) | (0.043) | (0.036) | (0.035) |
| South | −0.012 | 0.012 | 0.000 | 0.115*** | −0.089** | −0.026 |
| | (0.027) | (0.053) | (0.052) | (0.043) | (0.036) | (0.036) |
| Number of wage earners | −0.024** | 0.009 | 0.015 | 0.069*** | −0.026* | −0.044*** |
| | (0.011) | (0.017) | (0.017) | (0.017) | (0.014) | (0.015) |
| Observations | 726 | 726 | 726 | 726 | 726 | 726 |

Notes: Reference category for husband's age: younger than 35. Reference category for risk behavior: low propensity to take risks. Reference category for macro-regions: Center. Standard errors are in parenthesis. *** p<0.01, ** p<0.05, * p<0.10.

**Table 4. Linear-Mixed Model with Random Intercepts (726 families)**

| | Bargaining cost |
|---|---|
| Husband's share at the threat point > 0.5 | −0.944* |
| | (0.510) |
| Husband's market wage/wife's market wage | 2.088 |
| | (4.737) |
| Husband's decision-making power | 0.291** |
| | (0.136) |
| Husband inherited the farm activity | −0.218 |
| | (0.413) |
| Medium-highly educated husband | 0.477 |
| | (0.458) |
| Medium-highly educated wife | 1.396** |
| | (0.679) |
| Husband aged 35-59 | 1.995*** |
| | (0.676) |
| Husband aged 60 and over | 1.057 |
| | (0.831) |
| Medium propensity to take risks | 1.135** |
| | (0.472) |
| High propensity to take risks | 0.733 |
| | (0.466) |
| North | 3.369*** |
| | (0.540) |
| South | 1.148** |
| | (0.544) |
| Number of wage earners | 0.912*** |
| | (0.175) |
| Agreement | −0.091*** |
| | (0.003) |
| Observations | 1,255,878 |
| Number of groups | 726 |

Notes: Reference category for husband's age: younger than 35. Reference category for risk behavior: low propensity to take risks. Reference category for macro-regions: Center. Standard errors are in parenthesis. *** p<0.01, ** p<0.05, * p<0.10.

**Appendix A - Proofs**

Proof of Proposition 1: Consider a point $(x^*, y^*, z^*, U^*)$ that satisfies equations (2) and (3). Assume that the allocation $(x^*, y^*, z^*)$ is not efficient. It means that there exists another feasible allocation $(x^0, y^0, z^0)$ such that $u_i(x_0^0, x_i^0, z^0) \geq u_i(x_0^*, x_i^*, z^*)$ for all $i \in N$ and $u_j(x_0^0, x_j^0, z^0) > u_j(x_0^*, x_j^*, z^*)$ for some individual $j \in N$. Let $e_i(p_i, x_0, z, U_i^*) \equiv Min_{x_i} \{p_i\, x_i: u_i(x_0, x_i, z) \geq U_i^*, x_i \in X_i\}, i \in N$. Assume that (3) holds. Under assumptions A1 (continuity), A2 (non-satiation) and A4 (no destitution), $u_i(x_0^0, x_i^0, z^0) \geq U_i^*$ implies that $e_i(p_i, x_0^0\,, z^0, U_i^*) \leq p_i\, x_i^0, i \in N$, and $u_j(x_0^0, x_j^0, z^0) > U_j^*$ implies that $e_j\left(p_j, x_0^0\,, z^0, U_j^*\right) < p_j\, x_j^0$. It follows that $q\, y^0 - p_0\, x_0^0 - \sum_{i \in N} p_i\, x_i^0 < q\, y^0 - p_0\, x_0^0 - \sum_{i \in N} e_i\,(p_i, x_0^0, z^0, U_i^*)$. But this contradicts (2). Now assume that (2) holds. The feasibility of $(x^0, y^0, z^0)$ implies that that it satisfies the budget constraint (1): $q\, y^0 - p_0\, x_0^0 - \sum_{i \in N} p_i\, x_i^0 \geq 0$. It follows that $0 \leq q\, y^0 - p_0\, x_0^0 - \sum_{i \in N} p_i\, x_i^0 < q\, y^0 - p_0\, x_0^0 - \sum_{i \in N} e_i\,(p_i, x_0^0, z^0, U_i^*) \leq Max_{x_0, z}\{q\, y - p_0 x_0 - \sum_{i \in N} e_i\,(p_i, x_0, z, U_i^*) : x_0 \in X_0, (y, z) \in F\} = W(p, q, U^*)$, which contradicts (3). Thus, equations (2) and (3) imply efficiency.

Now consider a feasible allocation $(x^a, y^a, z^a)$ that is efficient. Assume that it does not satisfy equations (2) and (3). This can happen in two ways. First, it can happen if $(x^a, y^a, z^a, U^a)$ satisfies equation (3) but not (2), i.e. if conditional on $U^a$ there exists a feasible allocation $(x^b, y^b, z^b) \neq (x^a, y^a, z^a)$ such that $D \equiv \left[qy^b -\ p_0 x^b - \sum_{i \in N} p_i\, x_i^b\right] - [qy^a -\ p_0 x^a - \sum_{i \in N} p_i\, x_i^a] > 0$. Under assumption A2, redistributing the income $D > 0$ to the $n$ individuals can make at least one person better without making anyone worse off, contradicting efficiency. Second, it can happen if $(x^a, y^a, z^a, U^a)$ satisfies equation (2) but not (3), i.e. if $U^a$ satisfies $W(p, q, U^a) = [qy^a - p_0 x_0^a - \sum_{i \in N} p_i\, x_i^a] \neq 0$. The household budget constraint (1) implies that $W(p, q, U^a) \geq 0$. But under assumption A2, having $W(p, q, U^a) > 0$ would mean that the income $W(p, q, U^a) > 0$ can be redistributed to the $n$ individuals to make at least one person better off without making anyone worse off, contradicting efficiency. It follows that efficiency implies equations (2) and (3).

Proof of Proposition 2: When $N^k \neq N$, equations (12a)-(12b) in S3b imply that $M^{k+1} < M^k$, i.e. that the maximum willingness-to-pay to avoid bargaining failure declines across iterations from $k$ to $k+1$. Thus, the bargaining process given in S moves in the direction of equalizing the willingness to accept a bargaining failure. Upon convergence, from S3a, the iterative scheme in S leads to a point where $N^{k^\#} = N$. From Definition 2, this identifies $(x^{k^\#}, y^{k^\#}, z^{k^\#})$ as a bargaining agreement.

Proof of Proposition 3: From Proposition 2, $(x^{k^*}, y^{k^*}, z^{k^*})$ is a bargaining agreement. Upon convergence, condition $S^E$ 3a states that $W(p, q, U^{k^*}) = 0$. Under assumptions A1-A4, Proposition 1 implies that $U^{k^*}$ is on the Pareto utility frontier. This implies that the allocation $(x^{k^*}, y^{k^*}, z^{k^*})$ is a bargaining agreement that is Pareto efficient. We now show that this agreement is a unique point on the Pareto utility frontier. Under assumption A2, note that $W(p, q, U)$ is strictly decreasing in $U$ and $e_i(p_i, x_0, z, U_i)$ is strictly increasing in $U_i, i \in N$. Assume that there are two different agreements on the Pareto utility frontier satisfying $U^a \neq U^b$, with $W(p, q, U^a) = 0$ and $W(p, q, U^b) = 0$. The function $W(p, q, U)$ being strictly decreasing in $U$ implies that there exists some $j \neq i \in N$ such that $U_i^a > U_i^b$ and $U_j^a < U_j^b$. Because the expenditure function $e_i(p_i, x_0, z, U_i)$ is strictly increasing in $U_i$, then $e_i(p_i, x_0, z, U_i^a) > e_i(p_i, x_0, z, U_i^b)$ and $e_j\left(p_j, x_0, z, U_j^a\right) < e_j(p_j, x_0, z, U_j^b)$. But this contradicts the definition of a bargaining agreement in equations (8)-(10), implying that only one point on the Pareto utility frontier can be a bargaining agreement.

Proof of Proposition 4: Under assumption A2, maximization with respect to $U = (U_1, \dots, U_n)$ implies that the budget constraint in equation (16) is necessarily binding. In turn, this implies equations (3), (5), (6) and (7). From Corollary 1, it follows that the solution to problem (16) is Pareto efficient and that $U^e$ is on the Pareto utility frontier: $U^e \in \{U: W(p, q, U) = 0\}$. First, consider when $W(p, q, U^t) = 0$. Then $U^t$ is on the Pareto frontier and $U^e = U^t$. Second, consider when $W(p, q, U^t) > 0$. Then, the maximization problem in (16) can be alternatively written as

$$\underset{x_0, y, z, U}{\text{Max}} \{ \textstyle\sum_{i \in N} \ln [e_i(p_i, x_0, z, U_i) - e_i(p_i, x_0, z_0, U_i^t)] : \sum_{i \in N} e_i(p_i, x_0, z, U_i) + p_0 x_0 = qy;\ x_0 \in X_0, (y, z) \in F \}. \quad (16')$$

Under assumption A2, $e_i(p_i, x_0, z, U_i)$ is strictly increasing in $U_i, i \in N$. Denoting by $\lambda$ the Lagrange multiplier associated with the budget constraint in (16'), the first-order necessary conditions with respect to $U_i$ in (16') give $\lambda = 1/[e_i(p_i, x_0, z, U_i) - p_i x_i^t], i \in N$. This implies that the solution to problem (16') satisfies the convergence criterion in $S^E$3a, i.e., that the solution to problem (16) is a bargaining agreement located on the Pareto utility frontier.

## Appendix B: Model Specification

This appendix describes the specification of the collective household model (Matteazzi, Menon and Perali 2017) adopted in the empirical analysis. Family members consume and produce both marketable and non-marketable goods. The set of market inputs is $v$ = {chemicals, materials, hired labor}, $H$ is family labor treated as a quasi-fixed factor, and the set of outputs is $s$ = {crop, livestock, milk, fruits, olives and grapes}. The set of consumption goods is $k$ = {leisure, domestic good, food, clothing, other market goods}. Member 1 is the wife and member 2 the husband.

Cost Function of Marketable Production

Matteazzi, Menon and Perali (2017) estimate the agricultural production technology from the dual side to account both for the non-homogeneity of family and hired labor and the fact that hired labor is a variable factor with an associated observable market wage, while family labor is a quasi-fixed factor with an associated shadow wage. The total restricted cost function takes a Translog form

$$\log VC = \alpha_0 + \sum_s \alpha_s \log q_s + \sum_f \beta_f \log r_f + \chi \log H + \frac{1}{2}\sum_s \sum_u \delta_{su} \log q_s \log q_u + \\ + \frac{1}{2}\sum_f \sum_v \gamma_{fv} \log r_f \log r_v + \sum_s \sum_f \rho_{sf} \log q_s \log r_f + \sum_f \xi_f \log H \log r_f \quad \text{(B1)}$$

where $VC$ denotes variable production costs, $r_f$ is the price of market input $f$, $H$ is family labor, $q_s$ is the level of the agricultural output $s$, and $\alpha_0$, $\alpha_s$, $\beta_f$, $\chi$, $\delta_{su}$, $\gamma_{fv}$, $\rho_{sf}$, $\xi_f$ are parameters. On-farm family labor $H$ is a quasi-fixed factor allocable both across activities, when information is available, and between spouses, as in our case. If entrepreneurs are minimizing costs, the quasi-fixed factor's shadow wage for family labor $w_{on}^*$ is derived by differentiating total costs with respect to the quasi-fixed factor

$$w_{on}^* = \left(\chi + \sum_{f=1}^{3} \xi_f \log r_f\right) \frac{VC}{H}, \quad \text{(B2)}$$

where $VC$ are the minimum variable costs expressed in level. Farm-household profits are

$$\pi = \sum_{s=1}^{4} p_{q_s} q_s - VC + TR. \quad \text{(B3)}$$

where $TR$ are the lump-sum income transfers that farm-households receive from the government.

Cost Function of the Nonmarketable Domestic Production

The total cost function for domestic production has a Translog form. The implicit price of the domestic good, $p_z^*$, is the exponent of a Translog unit cost function (Apps and Rees, 1997)

$$p_z^* = \exp\left(a_0 + \sum_{i=1}^{2} a_i \log w_i + \frac{1}{2}\sum_{i=1}^{2} \sum_{j=1}^{2} a_{ij} \log w_i \log w_j\right), \quad \text{(B4)}$$

where $w_i$ is spouse $i$ market wage and $a_0$, $a_i$, and $a_{ij}$ are parameters.

Individual Preferences

We adopt an AIDS specification for individual preferences

$$\omega_{ik} = \alpha_{ik} + t_{ik}(d_i) + \sum_n \nu_{ink} \log P_{ik} + \eta_{ik} \log\left(\frac{I_i}{A_i(P_{ik})}\right) \tag{B5}$$

Where $\omega_{ik}$ are the budget shares of individual $i$ for good $k$, $t_{ik}(d_i) = \sum_i \delta_{ik} \log d_{ik}$ is the $k$-th translating function with $d_i$ denoting demographic variables, $P_{ik}$ is the set of prices, $I_i$ is individual full income, and $A_i(P_{ik})$ is a Translog price index. $\alpha_0$, $\alpha_{ik}$, $\delta_{ik}$, $\nu_{ink}$, and $\eta_{ik}$ are parameters. Individual full income is the sum of the value of individual endowment of time with the individual share $\varphi_i$ of total household nonlabor income including profits from the marketable agricultural production (equation B3) and nonlabor income $y$,

$$I_i = w_i(l_i + t_i + L_i) + w_{on}^* h_i + \varphi_i, \tag{B6}$$

where $l_i$, $t_i$, $L_i$ and $h_i$ are spouse $i$'s time devoted to leisure, domestic work, market work and on-farm work. Spouses' shares of household nonlabor income are

$$\varphi_1 = \psi(w_1, w_2, p_1, p_2, y, d_f)(\pi + y), \tag{B7}$$

$$\varphi_2 = (\pi + y) - \varphi_1, \tag{B8}$$

where $(w_1, w_2)$ are the spouses' market wages, $(p_1, p_2)$ are clothing prices, $y$ is nonlabor income and $d_f$ distribution factors. The function $\psi$ is specified as a Cobb-Douglas

$$\psi = \left(w_1^{\theta_1} w_2^{\theta_2} p_1^{\theta_3} p_2^{\theta_4} y^{\theta_5} d_f^{\theta_6}\right) \in [0,1] \tag{B9}$$

with $\theta_5 = -\sum_{n=1}^{4} \theta_n$ to have individual income shares homogeneous of degree one in monetary variables and the consumption demands satisfying homogeneity of degree zero in prices and nonlabor income (Matteazzi, Menon and Perali 2017, Chavas *et al*. 2018). The price of the domestic good in equation B5 is derived as in equation B4. The individual private expenditure $e_i$ cannot exceed individual full income $e_i = \sum_k P_{ik} x_{ik} \leq I_i$ where $x_{ik}$ denotes the units of consumption goods. The individual indirect utility function is

$$V_i = \frac{\log e_i - \sum_k \alpha_{ik} \log P_{ik} - \sum_k t_{ik}(d) \log P_{ik} - 0.5 \sum_k \sum_n \nu_{ink} \log P_{in} \log P_{ik}}{\prod_k P_{ik}{}^{\eta_{ik}}}. \tag{B10}$$

## Appendix C - Implementation of the Bargaining Process $S^E$

In iterative scheme $S^E$, spouses negotiate the redistribution of household resources conditional on the household optimal production decisions.[1] They bargain over the amount of total household income available for the individual private consumptions of leisure, clothing, the domestically produced good, food and other goods. We mimic the iteration processes in $S^E$ using a stochastic simulation to approximate our theoretical analysis. At each iteration, the individual demands for the private goods, with the except of clothing, are randomly generated between a lower and an upper bound corresponding to the minimum and maximum consumption value observed in the dataset. For the individual demand of clothing, the simulation randomly chooses a value between a lower and an upper bound defined by equations (12a),[2] (12b)[3] and (15).[4] Therefore, at each iteration $k$ for each spouse, the individual private expenditure $e_i^k\left(p_i, x_0, z, u_i^k\left(x_0, x_i^k, z\right)\right)$ is randomly generated to ensure feasibility of the household budget.[5] At each iteration $k$, we calculate the willingness-to-pay $\Delta_i^k = e_i^k(p_i, x_0, z, u_i^k(x_0, x_i^k, z)) - e_i(p_i, x_0, z, U_i^t)$ of each spouse $i$. If the difference between the spouses' willingness-to-pay is sufficiently small,[6] spouses will reach an agreement; otherwise, they continue to negotiate until they reach the contract curve. We randomly generate a total of 1,000,000 feasible allocations.[7]

Table C1 shows the simulations of the bargaining scheme $S^E$ for the average family of our sample. Each row of the table represents a $k$-th iteration of the negotiation process. The first

[1] We predict household profits from the marketable agricultural production and the value of the non-marketable domestic good. With the addition of household non-labor income, we obtain household income ($\pi = qy$).

[2] For the household member with the highest willingness-to-pay to avoid the threat allocation, equation (12a) is $\Delta_i^{k+1} < \Delta_i^k$. Given $\Delta_i^k = e_i^k(p_i, x_0, z, u_i^k(x_0, x_i^k, z)) - e_i(p_i, x_0, z, U_i^t)$, where $e_i(p_i, x_0, z, U_i^t)$ is treated as given, from equation (12a) we have $e_i^{k+1}\left(p_i, x_0, z, u_i^{k+1}\left(x_0, x_i^{k+1}, z\right)\right) < e_i^k\left(p_i, x_0, z, u_i^k\left(x_0, x_i^k, z\right)\right)$. Hence, for the member with the highest willingness-to-pay, the expenditure at iteration $k$ is an upper bound for her private expenditure at $k + 1$.

[3] For the household member with the lowest willingness-to-pay to avoid the bargaining failure, equation (12b) implies that $e_i^{k+1}\left(p_i, x_0, z, u_i^{k+1}\left(x_0, x_i^{k+1}, z\right)\right) \leq e_j^k\left(p_j, x_0, z, u_j^k\left(x_0, x_j^k, z\right)\right) - e_j(p_j, x_0, z, U_j^t) + e_i(p_i, x_0, z, U_i^t)$.

[4] In presence of untapped efficiency gains at iteration $k$, equation (15) sets an increase in household total private expenditure at iteration $k + 1$ entailing a reduction in household welfare losses. For the member with the lowest willingness-to-pay to avoid the threat allocation, equation (15) implies that $e_i^{k+1}\left(p_i, x_0, z, u_i^{k+1}\left(x_0, x_i^{k+1}, z\right)\right) \geq e_i^k\left(p_j, x_0, z, u_i^k\left(x_0, x_i^k, z\right)\right) + e_j^k\left(p_j, x_0, z, u_j^k\left(x_0, x_j^k, z\right)\right) - e_j^{k+1}\left(p_j, x_0, z, u_j^{k+1}\left(x_0, x_j^{k+1}, z\right)\right)$.

[5] Given that the individual willingness-to-pay to avoid the threat allocation must be non-negative, the private expenditure associated with the threat allocation is a lower bound for spouses' expenditure on private goods. This means that, at each iteration and for all household members, $e_i^k(p_i, x_0, z, u_i^k(x_0, x_i^k, z)) \geq e_i(p_i, x_0, z, U_i^t)$.

[6] The difference $\Delta_i^k - \Delta_j^k$ is assumed to be sufficiently small if it is between −5€ and +5€ corresponding to about 0.06% of average household income.

[7] About 5% of the generated allocations are not feasible because total household expenditures exceed total household income. This event is especially frequent for households with low levels of household income.

iteration, $k = 1$, is the starting point of the negotiation processes, which corresponds to the sample average value. As previously mentioned, the threat point is defined as the level of private expenditure of a single person household differentiated by gender.[8] Spouses have a similar purchasing power at the threat point amounting to 1622€ for the wife and 1533€ for the husband. The equality of the individual purchasing power translates into an egalitarian distribution of resources between the spouses (at the threat point $\sigma_1 = 0.51$ and $\sigma_2 = 0.49$).

At the sample mean and given the threat point, the willingness-to-pay $\Delta_1^1$ of the wife to avoid the threat allocation is greater than the husband willingness-to-pay $\Delta_2^1$ (695€ and 657€, respectively). The wife is thus less willing to accept a bargaining failure than her husband and she will be willing to make a concession at $k = 2$ by reducing her willingness to face a bargaining failure. Comparing the result obtained at $k = 1$ with those at $k = 2$, we note a reduction in the private expenditure $e_1^k$ of the wife, while the husband private expenditure $e_2^k$ slightly increases. At $k = 2$ the wife willingness-to-pay $\Delta_1^2$ decreases and the willingness-to-pay of the husband $\Delta_2^2$ increases. Because the spouses have different willingness-to-pay, the negotiation continues until they reach an agreement. For the scheme S, a bargaining agreement is reached at $k = 54$, where the difference between the spouse willingness-to-pay is sufficiently small, $\Delta_1^{54} - \Delta_2^{54} = 4.4$. Note that the maximized net income ($W$) is not entirely allocated among the spouses, the family saves 423€, and this agreement is not Pareto efficient. In general, a Pareto efficient agreement requires more negotiations. As shown in the table, the efficient agreement is reached at $k = 136{,}526$. The difference between spouses' willingness-to-pay is sufficiently small and $W$ is almost 0.

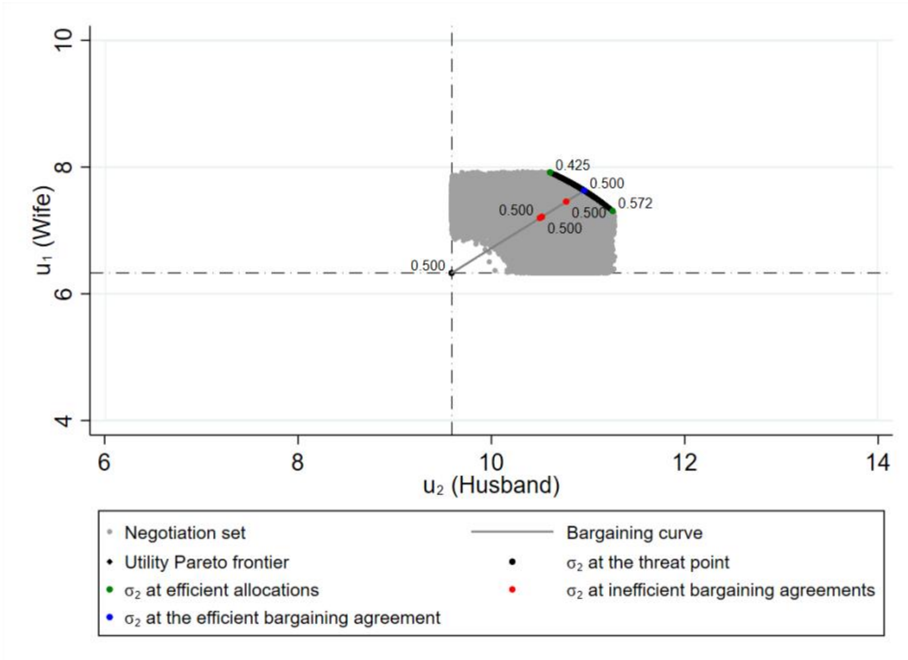
[8] The estimation of the cost of living of a single is based on the notion of indifference scales. Define the indifference scale as the index $I_i^h = e_i^s(p_i, x_0, z, U_i^T)/\pi(q, z)$ that adjusts the income of member *i* of household *h* to reach the same indifference curve as when living alone, where $e_i^s(p_i, x_0, z, U_i^T)$ is the income that member *i* would need if living alone (that is at the threat point) to attain the same level of utility, and $\pi(q, z)$ is the household income (Lewbel, 2003; Chiappori, 2016). The individual income at the threat point for member *i* is obtained from the knowledge of household expenditure $e^s(p, x_0, z, U^T)$ multiplied by the ratio of the actual share of resources within the couple $s_i$ with respect to an equal division, that is $e_i^s(p_i, x_0, z, U_i^T) = e^s(p, x_0, z, U^T) * (s_i/0.5)$. As an example, suppose that the cost of living of a reference couple is 1000€. Then, the cost of living of an equivalent adult is 500€. Using the estimates in Perali (1999) and Menon and Perali (2010) for Italy, the cost of living of a person living alone is 70% of the cost of living of a couple, then the cost of living of the single is 700€, which is 1.4 times the cost of an adult equivalent. Assume further that $s_1 = 0.6$ and $s_2 = 1 - s_1 = 0.4$, then $e_1^s$ and $e_2^s$ amount to 840€ and 560€, respectively.

**Table C1: Empirical Implementation of the Bargaining Scheme $S^E$ (Iterative Process)**

| | Wife | | | | | | Husband | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Expenditure for: | | | $e_1^k$ | $U_1^k$ | $\Delta_1^k$ | Expenditure for: | | | $e_2^k$ | $U_2^k$ | $\Delta_2^k$ | $W$ | $\sigma_2$ |
| $K$ | Leisure | Other goods | Clothing | | | | Leisure | Other goods | Clothing | | | | | |
| 1 | 1037.10 | 1272.81 | 7.28 | 2317.19 | 7.16 | 695.41 | 909.70 | 1272.81 | 6.84 | 2189.34 | 10.34 | 656.86 | 1313.87 | 0.49 |
| 2 | 230.21 | 1842.48 | 152.34 | 2225.03 | 7.08 | 603.26 | 185.59 | 1842.48 | 199.82 | 2227.89 | 10.38 | 695.41 | 1367.48 | 0.50 |
| 3 | 1165.07 | 839.24 | 312.87 | 2317.19 | 7.16 | 695.41 | 259.52 | 839.24 | 1021.78 | 2120.55 | 10.27 | 588.06 | 1382.67 | 0.48 |
| 4 | 396.84 | 1840.82 | 9.52 | 2247.18 | 7.10 | 625.40 | 1092.84 | 1840.82 | 0.00 | 2933.66 | 10.98 | 1401.18 | 639.55 | 0.57 |
| 5 | 1432.12 | 988.80 | 33.80 | 2454.72 | 7.28 | 832.94 | 860.87 | 988.80 | 1018.79 | 2868.46 | 10.93 | 1335.98 | 497.22 | 0.54 |
| … | | | | | | | | | | | | | | |
| 54 | 787.71 | 958.94 | 997.92 | 2744.57 | 7.51 | 1122.79 | 1267.27 | 958.94 | 426.75 | 2652.97 | 10.76 | 1120.48 | 422.86 | 0.49 |
| … | | | | | | | | | | | | | | |
| 136526 | 523.60 | 1148.80 | 1280.94 | 2953.34 | 7.66 | 1331.56 | 321.74 | 1148.80 | 1396.41 | 2866.94 | 10.93 | 1334.46 | 0.12 | 0.49 |

Notes: $e_i^k$ is the total private expenditure of member $i$ at iteration $k$, $U_i^k$ is the associated utility level, $\Delta_i^k$ is the individual willingness-to-pay to avoid the disagreement point allocation, $W$ is the measure of inefficiency, and $\sigma_2$ is the husband resource share.

**Figure C1: Egalitarian Threat Point**




Note: $\sigma_2$ is husband resource share.

## Appendix D - Additional Tables and Figures

**Table D1. Simulated Allocations, Bargaining Agreements, Efficient Allocations and Efficient Bargaining Agreements (726 Families)**

| Number of randomly generated allocations for each family | Number of families reaching | | |
|---|---|---|---|
| | at least one bargaining agreement | at least one efficient allocation | an efficient bargaining agreement |
| 5000 | 670 | 303 | 77 |
| 4000 | 670 | 301 | 77 |
| 3000 | 670 | 300 | 67 |
| 2000 | 670 | 291 | 57 |
| 1000 | 670 | 283 | 37 |
| 800 | 670 | 277 | 33 |
| 500 | 668 | 268 | 22 |
| 400 | 663 | 262 | 19 |
| 300 | 645 | 253 | 16 |
| 200 | 605 | 239 | 11 |
| 150 | 553 | 224 | 9 |
| 100 | 466 | 197 | 5 |
| 50 | 292 | 142 | 2 |

**Table D2. Descriptive Statistics (726 Families)**

| | **Mean** | **Std. Dev.** |
|---|---|---|
| **Outcome variables** | | |
| Share of families that reached a bargaining agreement | 0.92 | 0.27 |
| Share of families that reached an efficient allocation | 0.40 | 0.49 |
| Share of families that reached an efficient bargaining agreement | 0.08 | 0.27 |
| *Among families that reached a bargaining agreement, share of families that reached the outcome with a number of iterations* | | |
| ≤ 50 (*fast* families) | 0.44 | 0.50 |
| > 50 (*slow* families) | 0.56 | 0.50 |
| *Among families that reached an efficient allocation, share of families that reached the outcome with a number of iterations* | | |
| ≤ 50 (*fast* families) | 0.49 | 0.50 |
| > 50 (*slow* families) | 0.51 | 0.50 |
| **Control variables** | | |
| Husband's share at the threat point > 0.5 | 0.18 | 0.39 |
| Husband's market wage/Wife's market wage | 1.03 | 0.05 |
| Husband's decision-making power | 4.68 | 1.45 |
| Husband inherited the farm activity | 0.66 | 0.47 |
| Medium-highly educated husband | 0.52 | 0.50 |
| Medium-highly educated wife | 0.13 | 0.33 |
| Husband aged 35-59 | 0.73 | 0.44 |
| Husband aged 60 and over | 0.17 | 0.37 |
| Medium propensity to take risks | 0.28 | 0.45 |
| High propensity to take risks | 0.31 | 0.46 |
| North | 0.39 | 0.49 |
| South | 0.37 | 0.48 |
| Number of wage earners | 2.57 | 1.76 |